Ch-1 Complex Numbers.

Complex Number. :- $x^2+1=0$, $x^2+4=0$ are not salvable in R.

Euler was the first mathematician to introduct the ~~nur~~ symbal i (iota) far the square root of -1.

i.e. $i = \sqrt{-1}$

$i^2 = -1$

$i^3 = i^2 \cdot i = (-1)(i) = -i$

$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$

$$\boxed{\begin{aligned} i &= \sqrt{-1} \\ i^2 &= -1 \\ i^3 &= -i \\ i^4 &= 1 \end{aligned}}$$

Definition :- If a and b are two real number then a number af the farm a+ib is called Complex Number.

For eg $5+2i$, $-1+i$, $0+2i$, $1+0i$, are complex Number

$$Z = \underset{\substack{\downarrow \\ \text{Real part}}}{a} + i\underset{\substack{\searrow \\ \text{imaginary part}}}{b}$$

Q.1 Find i^{22}

salution.

$$i^{22} = (i^4)^5 \cdot i^2$$

$$= (1)^5 (-1) = (1)(-1) = -1$$

$4\,)\,22\,(\,5$
20
2

Q.2 find i^{33}

salution.

$$i^{33} = (i^4)^8 \cdot i^1$$

$$= (1)^8 \cdot (i)$$

$$= 1 \times i = i$$

$4\,)\,33\,(\,8$
32
1

Q.3 Find i^{47}

Salution.

$$i^{47} = (i^4)^{11} \cdot i^3$$

$$= (1)^{11} \cdot (-i)$$

$$= (1)(-i) = -i$$

$4\,)\,47\,(\,11$
4
7
4
3

Q.4 find i^{64}

Salution

$$i^{64} = (i^4)^{16} = (1)^{16} = 1$$

$4\,)\,64\,(\,16$
64
0

Q.5 find i^{236}

solution $i^{236} = (i^4)^{59} = (1)^{59} = 1$

Q.6 find i^{-99}

solution $\frac{1}{i^{99}} = \frac{1}{(i^4)^{24}\cdot i^3} = \frac{1}{(1)^{24}\cdot i^3}$

$= \frac{1}{i^3} = \frac{1}{-i} = \frac{1}{-i}\times\frac{i}{i}$

$= \frac{i}{-i^2} = \frac{i}{-(-1)} = \frac{i}{1} = i$

Q.7 find i^{-45}

solution $\frac{1}{i^{45}} = \frac{1}{(i^4)^{11}\cdot i} = \frac{1}{(1)^{11}\cdot i} = \frac{1}{i}$

$\frac{1}{i} = \frac{1}{i}\times\frac{i}{i} = \frac{i}{+i^2} = \frac{i}{-1} = -i$

Q.8 find $1+i^{20}+i^{30}$

Salution. $1+(i^4)^5+(i^4)^7\cdot i^2$

$1+(1)^5+(1)^7\cdot i^2$

$1+1+i^2 = 1+1-1 = 1$ Ans.

Q.9 Find $i^{15}+i^{25}+i^{35}$

Salation. $(i^4)^3\cdot i^3 + (i^4)^6\cdot i + (i^4)^8\cdot i^3$

$1\cdot i^3 + 1\cdot i + 1\cdot i^3 = i^3 + i + i^3$

$-i + \cancel{i} - \cancel{i} = -i$ Ans.

Q.10. $i^{33}+i^{47}+i^{54}$

$(i^4)^8\cdot i + (i^4)^{11}\cdot i^3 + (i^4)^{13}\cdot i^2$

$1\cdot i + 1\cdot i^3 + 1\cdot i^2 = i + i^3 + i^2$

$\cancel{i} - \cancel{i} - 1 = -1$ Ans.

Q.11 conjugate af complex Number.

$z = a + ib$

$\overline{z} = a - ib.$ $\left[\because \text{conjugate af } z = \overline{z}\right]$

Q.1 Find conjugate af $z = 2+3i$

Saluton $z = 2+3i$

$\overline{z} = 2-3i$

Q.2 find conjugate of $4-5i$.

Salution.

$$z = 4-5i$$

$$\bar{z} = 4+5i$$

Q.3 find conjugate of $\frac{1}{2+3i}$

Saluation.

$$z = \frac{1}{2+3i} \times \frac{2-3i}{2-3i}$$

$$z = \frac{2-3i}{(2)^2-(3i)^2} = \frac{2-3i}{4-9i^2}$$

$$z = \frac{2-3i}{4-9(-1)} = \frac{2-3i}{4+9}$$

$$z = \frac{2-3i}{13} = \frac{2}{13} - \frac{3i}{13}$$

$$\bar{z} = \frac{2}{13} + \frac{3}{13}i$$ Ay

Q.4 find conjugate of $\frac{1}{1+2i}$

Solution.

$$z = \frac{1}{1+2i} \times \frac{1-2i}{1-2i}$$

$$z = \frac{1-2i}{1^2-4i^2} = \frac{1-2i}{1-4(-1)}$$

$$z = \frac{1-2i}{1+4} = \frac{1-2i}{5}$$

$$z = \frac{1}{5} - \frac{2}{5}i$$

$$\overline{z} = \frac{1}{5} + \frac{2}{5}i$$

Q.5 find conjugate of $z = 2$

Solution

$$z = 2$$

$$\overline{z} = 2$$

Q.6 find conjugate of $z = 4i$

Solution

$$z = 4i$$

$$\overline{z} = -4i$$

Addition of complex Number

Q.1 If $z_1 = 2+3i$, $z_2 = 3+4i$

find $z_1 + z_2$

Salution. $z_1 + z_2 = (2+3i) + (3+4i)$

$= 5 + 7i$

Q.2 If $z_1 = 3-4i$, $z_2 = 5+4i$, find z_1+z_2

solution $z_1 + z_2 = 3-4i + 5 + 4i$

$= 8$

Subtraction of Complex Number

Q.3 If $z_1 = 8-4i$, $z_2 = 3+4i$, find $z_1 - z_2$

Salution $z_1 - z_2 = (8-4i) - (3+4i)$

$= 8-4i-3-4i$

$= 5-8i$ Ans

Q.4 If $z_1 = 5+3i$, $z_2 = 2+2i$

find $z_1 - z_2$

Salution $z_1 - z_2 = 5+3i - 2 - 2i$

$= 3+i$

Multiplication of complex Number.

Q.5 If $z_1 = (2-3i)$, $z_2 = 3+4i$

find $z_1 z_2$

Solution $z_1 z_2 = (2-3i)(3+4i)$

$= 6+8i-9i-12i^2$

$= 6-i-12(-1)$

$= 6-i+12 = 18-i$ Ans.

Q.6 If $z_1 = 3+i$, $z_2 = 2-i$ find $z_1 \cdot z_2$

Solution $z_1 z_2 = (3+i)(2-i)$

$= 6-3i+2i-i^2$

$= 6-i+1$

$= 7-i$

Q.7 If $z_1 = 4+i$, $z_2 = 2+3i$

find $z_1 \cdot z_2$

Solution $z_1 z_2 = (4+i)(2+3i)$

$= 8+12i+2i+3i^2$

$= 8+14i-3$

$= 5+14i$ Ans.

Conjugate of complex Number

Find the conjugate of $\frac{(3+2i)^2}{3+i}$

solution. $z = \frac{(3+2i)^2}{3+i}$

$$z = \frac{9+4i^2+12i}{3+i} = \frac{9-4+12i}{3+i}$$

$$= \frac{5+12i}{3+i} \times \frac{3-i}{3-i}$$

$$= \frac{15-5i+36i-12i^2}{9-i^2}$$

$$z = \frac{15+31i+12}{9+1} = \frac{27+31i}{10}$$

$$\overline{z} = \frac{27}{10} - \frac{31}{10} i$$

Modulas of Complex Number

if $z = a+ib$

then modulas of z is defined as

$$|z| = \sqrt{a^2+b^2}$$

Q.1 Find modulas of complex Number

$2+3i$

Salution $z = 2+3i = a+bi$

Here $a=2$, $b=3$

$|z| = \sqrt{a^2+b^2} = \sqrt{(2)^2+(3)^2}$

$= \sqrt{4+9} = \sqrt{13}$

Q.2 Find modulas of ① $2-i$ (ii) $3+2i$ (iii) $1-i$

Salution ① Here $z = 2-i = a+bi$

$a=2$, $b=-1$

$|z| = \sqrt{a^2+b^2}$

$= \sqrt{(2)^2+(-1)^2}$

$= \sqrt{4+1} = \sqrt{5}$

(ii) $z = 3-2i = a+bi$

$a=3$, $b=-2$

$|z| = \sqrt{(3)^2+(-2)^2}$

$= \sqrt{9+4} = \sqrt{13}$

(iii) $z = 1-i = a+bi$

$a=1$, $b=-1$

$|z| = \sqrt{a^2+b^2} = \sqrt{(1)^2+(-1)^2} = \sqrt{1+1} = \sqrt{2}$ Ans

Q.3 find modulas af complex Number.

$$(1+2i)(2-3i)$$

Solution. $z = (1+2i)(2-3i)$

$= 2+4i-3i-6i^2$

$= 2+i+6 = 8+i$

Here $a = 8$, $b = 1$

$|z| = \sqrt{(8)^2+(1)^2} = \sqrt{64+1} = \sqrt{65}$

Q.4. Find modulas af complex Numbers.

$$\frac{1}{3+2i}$$

Solution.

$z = \frac{1}{3+2i} \times \frac{3-2i}{3-2i} = \frac{3-2i}{9-4i^2}$

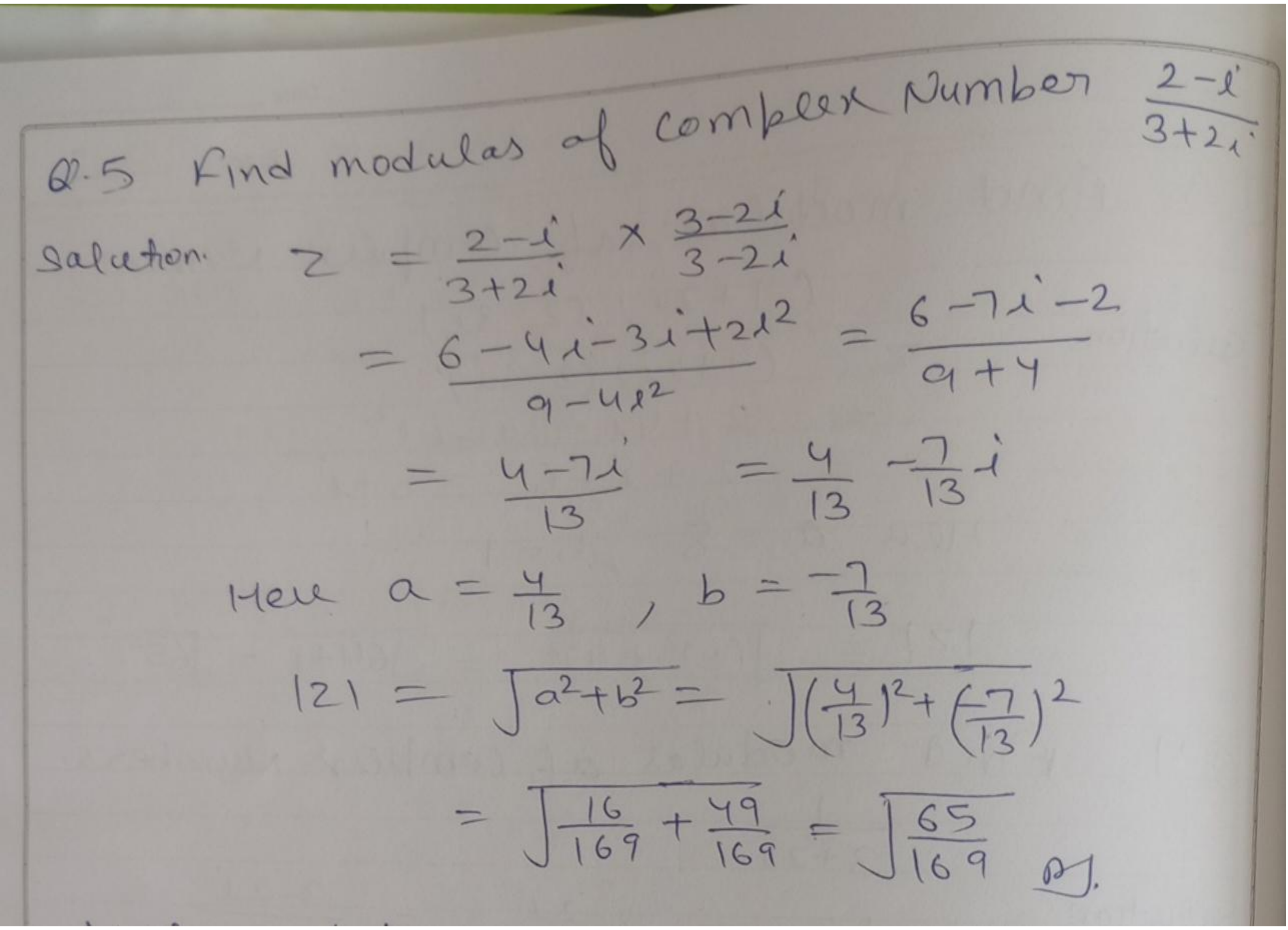
$z = \frac{3-2i}{9+4} = \frac{3-2i}{13}$

$z = \frac{3}{13} - \frac{2}{13}i$

$a = \frac{3}{13}$, $b = \frac{-2}{13}$

$|z| = \sqrt{\left(\frac{3}{13}\right)^2 + \left(\frac{-2}{13}\right)^2}$

$= \sqrt{\frac{9}{169} + \frac{4}{169}} = \sqrt{\frac{13}{169}} = \sqrt{\frac{1}{13}} = \frac{1}{\sqrt{13}}$ Ans



Q.5 Find modulas of complex Number $\frac{2-i}{3+2i}$

Salution. $z = \frac{2-i}{3+2i} \times \frac{3-2i}{3-2i}$

$= \frac{6-4i-3i+2i^2}{9-4i^2} = \frac{6-7i-2}{9+4}$

$= \frac{4-7i}{13} = \frac{4}{13} - \frac{7}{13}i$

Here $a = \frac{4}{13}$, $b = \frac{-7}{13}$

$|z| = \sqrt{a^2+b^2} = \sqrt{\left(\frac{4}{13}\right)^2 + \left(\frac{-7}{13}\right)^2}$

$= \sqrt{\frac{16}{169} + \frac{49}{169}} = \sqrt{\frac{65}{169}}$ Ans.

Q. 1 Express the fallowing complex number in standard farm a+ib

(i) $\frac{3+2i}{2-i}$ (ii) $\frac{(2-3i)(3+i)}{4+i}$

Solution.

$$z = \frac{3+2i}{2-i} \times \frac{2+i}{2+i} = \frac{6+3i+4i+2i^2}{4-i^2}$$

$$= \frac{6+7i-2}{4+1} = \frac{4+7i}{5}$$

$$= \frac{4}{5} + \frac{7}{5}i = a+ib$$ Ans.

(ii) $\frac{(2-3i)(3+i)}{4+i}$

salution

$$z = \frac{(2-3i)(3+i)}{4+i} = \frac{6-9i+2i-3i^2}{4+i}$$

$$= \frac{6-7i+3}{4+i} = \frac{9-7i}{4+i} \times \frac{4-i}{4-i}$$

$$= \frac{36-9i-28i+7i^2}{16-i^2}$$

$$= \frac{36-37i-7}{16+1} = \frac{29-37i}{17}$$

$$= \frac{29}{17} - \frac{37}{17}i = a+ib$$

Ans.

imp 4 marks

Polar farm of complex Number.

If $z = a+ib$, then its Polar farm is given by

$$z = r(\cos\theta + i\sin\theta)$$

where $r = |z| = \sqrt{a^2+b^2}$

$$\tan\theta = \frac{b}{a}$$

Q.1 Find Palar farm of complex Number

$$z = \sqrt{3} + i$$

Salution Here $z = \sqrt{3}+i = a+ib$

$$a = \sqrt{3}, \quad b = 1$$

$$r = |z| = \sqrt{(\sqrt{3})^2+1^2} = \sqrt{3+1} = \sqrt{4} = 2$$

Now $\tan\theta = \frac{b}{a}$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \tan 30^\circ \qquad \left[30^\circ = \frac{\pi}{6}\right]$$

$$\tan\theta = \tan\frac{\pi}{6}$$

$$\theta = \pi/6$$

Palar Farm of z is

$$z = r(\cos\theta + i\sin\theta)$$

$$z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

Q.2 Find Palar farm of complexnumber
$z = \sqrt{3} - i$

Salution. $z = \sqrt{3} - i$

$a = \sqrt{3}$, $b = -1$

$r = |z| = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{3+1} = \sqrt{4} = 2$

$\tan\theta = \frac{b}{a} = -\frac{1}{\sqrt{3}}$

$\tan\theta = -\tan\frac{\pi}{6}$

$\tan\theta = \tan\left(-\frac{\pi}{6}\right)$

$\theta = -\pi/6$

Palar farm is $z = r(\cos\theta + i\sin\theta)$

$z = 2\left[\cos(-\pi/6) + i\sin(-\pi/6)\right]$

Q.3 Find Palar farm of $z = -1 - i$

Salution. $z = -1 - i$, Here $a = -1$, $b = -1$

$r = |z| = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + (-1)^2}$

$= \sqrt{1+1} = \sqrt{2}$

$\tan\theta = \frac{b}{a} = \frac{-1}{-1} = 1$

$\tan\theta = 1 = \tan\pi/4 \Rightarrow \theta = \pi/4$

Palar farm is $z = r(\cos\theta + i\sin\theta)$

$z = \sqrt{2}\,(\cos\pi/4 + i\sin\pi/4)$ Ans.

Teacher's Signature

Q.4 Find the polar farm of $1-\sqrt{3}i$

Salution $z = 1-\sqrt{3}i$

$a=1, \quad b=-\sqrt{3}$

$r = |z| = \sqrt{a^2+b^2} = \sqrt{(1)^2+(-\sqrt{3})^2}$

$= \sqrt{1+3} = \sqrt{4} = 2$

$\tan\theta = \frac{b}{a} = \frac{-\sqrt{3}}{1} = -\sqrt{3} = -\tan 60^\circ$

$\tan\theta = -\tan\frac{\pi}{3}$ $\quad \left[\because 60^\circ = \frac{\pi}{3}\right]$

$\tan\theta = \tan\left(-\frac{\pi}{3}\right)$

$\theta = -\pi/3$

Palar farm is $z = r(\cos\theta + i\sin\theta)$

$z = 2\left[\cos(-\pi/3) + i\sin(-\pi/3)\right]$

Q.5 Find the real values of x and y if

~~Solt~~ $(1-i)x + (1+i)y = 1-3i$

Saluton. $(1-i)x + (1+i)y = 1-3i$

$x - xi + y + yi = 1-3i$

$(x+y) + i(-x+y) = 1-3i$

comparing real and imaginary part both sides, we have

$x+y = 1$ —— ①

$-x+y = 3$ —— ②

Now, salue eq^n. ① and ②,

Adding ① and ②, we have

$2y = 4$

$y = \frac{4}{2} = 2$

Put $y = 2$ in ①

$x + 2 = 1 \Rightarrow x = 1-2 = -1$

Q6 find the real values of x and y if

$(2-3i)x + (3-i)y = 2+4i$

Solution

$(2-3i)x + (3-i)y = 2+4i$

$2x - 3ix + 3y - iy = 2+4i$

$(2x+3y) + i(-3x-y) = 2+4i$

comparing real and imaginary part both sides

$2x + 3y = 2$ —— ①

$-3x - y = 4$ —— ②

multiply ① by 3 and ② by 2, and then adding

$\cancel{6x} + 9y = 6$

$-\cancel{6x} - 2y = 8$

$7y = 14$

$y = 14/7 = 2$

Put $y = 2$ in ①

$$2x + 3(2) = 2$$
$$2x = 2 - 6 = -4 \Rightarrow x = -2 \text{ Ans.}$$

H.W. [redacted]

Q.1 Find Palar form of following complex number

① $1 + i$ (iv) $1 - i$

(ii) $-1 - i$ (v) $-1 - \sqrt{3}i$

(iii) $-\sqrt{3} + i$

Q.2. Find real values of x and y

(i) $(x + iy)(2 - 3i) = 4 + i$

(ii) $(2 + i)x + (3 - i)y = 2 - 2i$

Ans. 1 (i) $r = \sqrt{2}\,(\cos \pi/4 + i \sin \pi/4)$

(IV) $r = \sqrt{2}\,\left(\cos(-\pi/4) + i \sin(-\pi/4)\right]$

(V) $r = \sqrt{2}\,(\cos \pi/3 + i \sin \pi/3)$

(II) $r = \sqrt{2}\,\left[\cos(+\pi/4) + i \sin(+\pi/4)\right]$

(III) $r = 2\,\left[\cos(-\pi/6) + i \sin(-\pi/6)\right]$

Q. 2 (i) $x = 5/13,\ y = 14/13$

Q.1. Find the real values of x and y if
$$(x+y) + i(2x-y) = 3+2i$$

Solution
$$(x+y) + i(2x-y) = 3+2i$$
Comparing real and imaginary part both sides

$$x+y = 3 \quad — ①$$
$$2x - y = 2 \quad — ②$$

Adding ① and ②

$$3x = 5 \quad \Rightarrow x = 5/3$$

Put $x = 5/3$ in ①

$$\frac{5}{3} + y = 3 \quad \Rightarrow y = 3 - \frac{5}{3} = \frac{9-5}{3} = \frac{4}{3}$$

Q.2 Find the real values of x and y if
$$(1+i)x + (2-i)y = 2+3i$$

Solution
$$(1+i)x + (2-i)y = 2+3i$$
$$x + xi + 2y - iy = 2+3i$$
$$(x+2y) + i(x-y) = 2+3i$$
Comparing real and imaginary part both sides

$$x+2y = 2 \quad \text{—①}$$
$$x-y = 3 \quad \text{—②}$$

Subtract ② from ①

$$\begin{array}{r} \cancel{x}+2y = 2 \\ \cancel{x}-y = 3 \\ -\quad + \quad - \\ \hline 3y = -1 \end{array} \Rightarrow y = -1/3$$

Put $y = -1/3$ in ②

$$x + \frac{1}{3} = 3 \Rightarrow x = 3 - \frac{1}{3} = \frac{8}{3} \text{ Ans.}$$

Q.3 Find values of x and y if

$$(1+i)(x+iy) = 5+4i$$

Solution
$$x + xi + iy + i^2 y = 5+4i$$
$$x + xi + iy - y = 5+4i$$
$$(x-y) + i(x+y) = 5+4i$$
$$x - y = 5 \quad \text{—①}$$
$$x + y = 4 \quad \text{—②}$$

Adding ① and ②

$$2x = 9 \Rightarrow x = 9/2$$

Put $x = 9/2$ in ①

$$\frac{9}{2} - y = 5 \Rightarrow -y = 5 - \frac{9}{2} \Rightarrow \frac{1}{2} \Rightarrow y = -1/2$$

Multiplicative inverse of z is $\frac{1}{z}$

as $z \cdot \frac{1}{z} = 1$

Q.4 Find the multiplicative inverse of $2+3i$

solution Multiplicative inverse of $2+3i = \frac{1}{2+3i}$

$$\frac{1}{2+3i} \times \frac{2-3i}{2-3i}$$

$$= \frac{2-3i}{4-9i^2} = \frac{2-3i}{4+9} = \frac{2-3i}{13}$$ Ans

Q5 Find the multiplicative inverse of $(3-i)^2$

solution $(3-i)^2 = 3+i^2-6i = 3-1-6i$
$= 2-6i$

multiplicative inverse is $\frac{1}{2-6i}$

$$= \frac{1}{2-6i} \cdot \times \frac{2+6i}{2+6i}$$

$$= \frac{2+6i}{4-36i^2} = \frac{2+6i}{4+36}$$

$$= \frac{2+6i}{40} = \frac{2(1+3i)}{40}$$

$$= \frac{1+3i}{20}$$ Ans

Q.6 Find the square root of complex number $5+12i$

Salution. Let $\sqrt{5+12i} = a+ib$ —①

squaring both sides

$(\sqrt{5+12i})^2 = (a+ib)^2$

$5+12i = a^2+i^2b^2+2iab$

$5+12i = a^2-b^2+2iab$

comparing real and imaginary part both sides,

$a^2-b^2 = 5$ —②

$2ab = 12 \Rightarrow ab = \frac{12}{2} = 6$

$b = \frac{6}{a}$ —③

Put $b = \frac{6}{a}$ in ②

$a^2 - \left(\frac{6}{a}\right)^2 = 5$

$a^2 - \frac{36}{a^2} = 5 \Rightarrow \frac{a^4-36}{a^2} = 5$

$a^4 - 36 = 5a^2$

$a^4 - 5a^2 - 36 = 0$

$(a^2-9)(a^2+4) = 0$

$a^2 = 9$, $a^2 \neq -4$

$a = \pm\sqrt{9} = \pm 3$

Put $a = \pm 3$ in ②

$$b = \frac{6}{a} = \frac{6}{\pm 3} = \pm 2$$

Put values of a and b in

$$\sqrt{5+12i} = \pm 3 \pm 2i = \pm(3+2i)$$ Ay.

Q7 find square root of $-7-24i$

Solution let $\sqrt{-7-24i} = a+ib$ — ①

squaring both sides

$$(\sqrt{-7-24i})^2 = (a+ib)^2$$

$$-7-24i = a^2+i^2b^2+2iab$$

$$-7-24i = a^2-b^2+2iab$$ comparing real and imagina part

$$\Rightarrow a^2-b^2 = 7$$ — ②

$$2ab = -24$$

$$ab = -12 \Rightarrow b = \frac{-12}{a}$$ — ③

Put $b = -12/a$ in ②

$$a^2 - \left(\frac{-12}{a}\right)^2 = 7$$

$$a^2 - \frac{144}{a^2} = 7$$

$$\frac{a^4-144}{a^2} = 7$$

$a^4 - 144 = 7a^2$

$a^4 - 7a^2 - 144 = 0$

$a^4 - 16a^2 + 9a^2 - 144 = 0 \Rightarrow a^2(a^2-16) + 9(a^2-16) =$

$(a^2-16)(a^2+9) = 0$

$a^2 \neq -9$

$a^2 = 16$

$a = \pm 4$

Put $a = \pm 4$ in ③

$b = -\frac{12}{a} = \frac{-12}{\pm 4} = \pm 3$

Put values of a and b in ①

$\sqrt{-7-24i} = \pm 4 \pm 3i$ Ans

H.W

Q.1. Find values of x and y if

(1) $(1+i)(x+iy) = 2+i$

② $(2+i)x + (3+i)y = 3+4i$

Q.2 Find multiplicative inverse of

(I) $3-4i$ (II) $(3+2i)^2$

Q.3 Find square root of (I) $5-12i$

(II) $7+24i$

Chapter-2. Logarithms

Definition :- The logarithm of any number is defined as,

if a is a positive real number, other than 1 and x is a rational such that $a^x = N$, then logarithm of N to base a is x. It is written as

$$\log_a N = x$$

Thus if $\boxed{a^x = N \Leftrightarrow \log_a N = x}$

Note ① $a^x = N$ is called exponential farm and $\log_a N = x$ is called the logarithm farms.

(II) For e.g. ① $3^3 = 27$

$\Rightarrow \log_3 27 = 3$

(ii) $2^4 = 16$

$\log_2 16 = 4$

(iii) $4^3 = 64$

$\log_4 64 = 3$

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Note (2) The logarithm of a negative number to a positive base is note defined.

for e.g $\log_2(-3)$ is not defined

Note (3) Only positive real numbers are taken as bases.

four Standard Logarithms

1 $\log_a(1) = 0 \quad (a \neq 0)$

(2) $\log_a(a) = 1$

(3) $\log_a \infty = \infty \quad (a > 1)$

(4) $\log_a 0 = (-\infty)$,

Q.1 Evaluate $\log_5 125$

Solution. Let $\log_5 125 = x$

$\Rightarrow 5^x = 125$

$\Rightarrow 5^x = 5^3$

$\Rightarrow x = 3$

So $\log_5 125 = 3$

Q.2 Evaluate $\log_3 81$

Solution. Let $\log_3 81 = x$

$3^x = 81 = 3^4 \Rightarrow x = 4$

$\Rightarrow \log_3 81 = 4$

Q.3 Evaluate $\log_2 \frac{1}{64}$

Solution. Let $\log_2 \frac{1}{64} = x$

$2^x = \frac{1}{64} = \frac{1}{2^6}$

$2^x = 2^{-6}$

$\Rightarrow x = -6$

Hence $\log_2 \frac{1}{64} = -6$

Q.4 Evaluate $\log_8 64$

Salution. $\log_8 64 = x$

$8^x = 64$

$\Rightarrow 8^x = 8^2$

$\Rightarrow x = 2$

$\Rightarrow \log_8 64 = 2.$

⑪ $\log_3 243$

Let $\log_3 243 = x$

$3^x = 243$

$3^x = 3^5$

$x = 5$

$\Rightarrow \log_3 243 = 5$ Ans.

Q.5 $\log_{10}\sqrt{100}$

Salution. Let $\log_{10}\sqrt{100} = x$

$10^x = \sqrt{100}$

$10^x = 10$

$x = 1$

$\Rightarrow \log_{10}\sqrt{100} = 1$

Q.6 $\log_4 \sqrt[3]{64}$

Salution. $\log_4 \sqrt[3]{64} = x$

$4^x = \sqrt[3]{64}$

$4^x = \sqrt[3]{4\times4\times4}$

$4^x = 4$

$x = 1 \Rightarrow \log_4 \sqrt[3]{64} = 1$

Q.7 $\log_2 \sqrt{8}$

solution $\log_2 \sqrt{8} = x$

$2^x = \sqrt{8}$

$2^x = \sqrt{2^3} = 2^{3/2}$

$\Rightarrow x = 3/2$

$\Rightarrow \log_2 \sqrt{8} = \frac{3}{2}$ Ans

Q.1 Write the fallowing in farm of logarithm.

(1) $3^4 = 81$ (II) $2^4 = 16$ (III) $4^{3/2} = 8$

(IV) $(125)^{1/3} = 5$ (V) $2^{-7} = \frac{1}{128}$

Salution. (1) $3^4 = 81$

$\log_3 81 = 4$

(II) $2^4 = 16$

$\log_2 16 = 4$

(III) $4^{3/2} = 8$

$\log_4 8 = 3/2$

(IV) $(125)^{1/3} = 5$

$\log_{125} 5 = \frac{1}{3}$

(V) $2^{-7} = \frac{1}{128}$

$\log_2 \frac{1}{128} = -7$

22. Express each of following in exponential form

(i) $\log_3 81 = 4$ ⓘⓘ $\log_5 125 = 3$

(iii) $\log_7 343 = 3$ (iv) $\log_6 216 = 3$

(v) $\log_2 \frac{1}{16} = -4$ (vi) $\log_8 \frac{1}{64} = -2$

Solution ① $\log_3 81 = 4$

$\Rightarrow 3^4 = 81$ As.

ⓘⓘ $\log_5 125 = 3$

$\Rightarrow 5^3 = 125$ Ay

ⓘⓘⓘ $\log_7 343 = 3$

$\Rightarrow 7^3 = 343$

(iv) $\log_6 216 = 3$

$\Rightarrow 6^3 = 216$

(v) $\log_2 \frac{1}{16} = -4$

$\Rightarrow 2^{-4} = \frac{1}{16}$

(vi) $\log_8 \frac{1}{64} = -2$

$8^{-2} = \frac{1}{64}$ Ay.

Q.3(i) Find the value of $\log_2 \frac{\sqrt{64}}{\sqrt{8}}$

Solution $\log_2 \frac{\sqrt{64}}{\sqrt{8}} = \log_2 \frac{\sqrt{64}}{\sqrt{8}} = \log_2 \sqrt{8}$

Let $\log_2 \sqrt{8} = x$

$$2^x = \sqrt{8} \Rightarrow 2^x = \sqrt{2^3} = (2^3)^{1/2}$$

$$2^x = 2^{3/2} \Rightarrow x = 3/2$$

$\Rightarrow \log_2 \frac{\sqrt{64}}{\sqrt{8}} = \frac{3}{2}$ Ans

Q.3(ii) $\log_{10} \sqrt[3]{100}$, let $\log_{10} \sqrt[3]{100} = x$

$$10^x = \sqrt[3]{100}$$

$$10^x = \sqrt[3]{10^2} \Rightarrow 10^x = (10^2)^{1/3}$$

$$10^x = 10^{2/3} \Rightarrow x = 2/3$$

$\Rightarrow \log_{10} \sqrt[3]{100} = 2/3$

Properties of logarithm

(I) Product formula,

$$\log_a mn = \log_a m + \log_a n$$

(II) $$\log_a \frac{m}{n} = \log_a m - \log_a n$$

(III) $$\log_a m^n = n \log_a m$$

(IV) $$\log_b m = \frac{\log_a m}{\log_a b}$$

(I) Natural Logarithm :- Logarithms of the base 'e' is called natural logarithms.

$$\log_e m = \log m$$

(II) Common Logarithm - Logarithm of the base 10 is called common logarithm.

$$\log_{10} x \quad , \quad \log_{10} 200$$

Prove $\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243} = \log 2$

$7\log\frac{16}{15} + 5\log\frac{25}{24} + 3\log\frac{81}{80} = \log 2$

Example 3. Prove that :

(*i*) $7\log\frac{10}{9} - 2\log\frac{25}{24} + 3\log\frac{81}{80} = \log 2$

(*ii*) $\log_2\frac{11}{15} + \log_2\frac{490}{297} - 2\log_2\frac{7}{9} = \log_2 2 = 1$

Sol. (*i*) **L.H.S.**

$7\log\frac{10}{9} - 2\log\frac{25}{24} + 3\log\frac{81}{80}$

$= 7(\log 10 - \log 9) - 2(\log 25 - \log 24) + 3(\log 81 - \log 80)$

$= 7(\log(5 \times 2) - \log 3^2) - 2(\log 5^2 - \log(3 \times 2^3) + 3(\log 3^4 - \log(5 \times 2^4))$

$= 7(\log 5 + \log 2 - 2\log 3) - 2(2\log 5 - \log 3 - 3\log 2) + 3(4\log 3 - \log 5 - 4\log 2)$

$= 7\log 5 + 7\log 2 - 14\log 3 - 4\log 5 + 2\log 3 + 6\log 2 + 12\log 3 - 3\log 5 - 12\log 2$

$= \log 2 =$ R.H.S.

(*ii*) **L.H.S.**

$\log_2\frac{11}{15} + \log_2\frac{490}{297} - 2\log_2\frac{7}{9}$

$= (\log_2 11 - \log_2 15) + (\log_2 490 - \log_2 297) - 2(\log_2 7 - \log_2 9)$

$= (\log_2 11 - \log_2 (5 \times 3)) + (\log_2 (7^2 \times 5 \times 2) - \log_2 (3^3 \times 11)) - 2(\log_2 7 - \log_2 3^2)$

$= \log_2 11 - \log_2 5 - \log_2 3 + \log_2 7^2 + \log_2 5 + \log_2 2 - \log_2 3^3 - \log_2 11 - 2\log_2 7 + \log_2 3^2$

$= \log_2 11 - \log_2 5 - \log_2 3 + 2\log_2 7 + \log_2 5 + \log_2 2 - 3\log_2 3 - \log_2 11 - 2\log_2 7$
$+ 4\log_2 3$

$= \log_2 2 = 1 =$ R.H.S.

Example 4. (*i*) **If $a^2 + b^2 = 18ab$, show that $\log\frac{1}{4}(a-b) = \frac{1}{2}(\log a + \log b)$.**

(*ii*) **If $a^3 + b^3 = 0$ and $a + b \neq 0$, show that $\log(a+b) = \frac{1}{2}(\log a + \log b + \log 3)$.**

Sol. (*i*)

$$a^2 + b^2 = 18ab \text{ (given)}$$

$$a^2 + b^2 - 2ab = 18ab - 2ab$$

$$(a-b)^2 = 16\,ab$$

$$a - b = 4\sqrt{ab}$$

$$\frac{1}{4}(a-b) = (ab)^{\frac{1}{2}}$$

Taking log both sides

Chapter-8 Permutation and combination

Factorial (!)

Meaning of $n!$

$n! = n(n-1)(n-2)(n-3) - - - - \cdots\cdots 3.2.1$

for e.g. $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$4! = 4 \times 3 \times 2 \times 1 = 24$

$3! = 3 \times 2 \times 1 = 6$

$2! = 2 \times 1 = 2$

$1! = 1$

$$\boxed{0! = 1}$$

Here, we note that

$5! = 5 \times 4 \times 3 \times 2 \times 1$

$= 5 \times 4!$

or $5! = 5 \times 4 \times 3 \times 2 \times 1$

$= 5 \times 4 \times 3!$

and $4! = 4 \times 3 \times 2 \times 1$

$= 4 \times 3!$

Note: Factorial of negative number and proper fraction is not defined.

Exercise → 1.

Ex → 1 Evaluate $\frac{8!}{6!}$

Salution. $\frac{8!}{6!} = \frac{8\times7\times\cancel{6!}}{\cancel{6!}} = 56$

Ex → 2 Evaluate $\frac{5!}{2!}$

Salution. $\frac{5\times4\times3\times\cancel{2!}}{\cancel{2!}} = 60$

Ex → 3 Evaluate $\frac{6!}{4!\times3!}$

Salution. $\frac{6!}{4!\times3!} = \frac{\cancel{6}\times5\times\cancel{4}\times\cancel{3!}}{\cancel{4}\times\cancel{3}\times\cancel{2}\times1\times\cancel{3!}} = 5$

Ex → 4 Evaluate $\frac{8!}{6!\times2!}$

Salution. $\frac{8!}{6!\times2!} = \frac{\overset{4}{\cancel{8}}\times7\times\cancel{6!}}{\cancel{6!}\times\cancel{2}\times1} = 28$

Ex → 5 Evaluate $\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}$

Salution

$$\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!} = \frac{1}{2!}+\frac{1}{3\times2!}+\frac{1}{4\times3\times2!}$$

$$= \frac{1}{2!}\left(1+\frac{1}{3}+\frac{1}{12}\right) = \frac{1}{2\times1}\left(\frac{12+4+1}{12}\right)$$

$$= \frac{1}{2}\left(\frac{17}{12}\right) = \frac{17}{24}$$ Ay.

Ex→6. which of the fallowing are true

(1) $(3+4)! = 3! + 4!$

Salution. L.H.S $(3+4)! = 7! = 7\times6\times5\times4\times3\times2\times1$
$= 5040$

R.H.S $3! + 4!$

$= 3\times2\times1 + 4\times3\times2\times1 = 6+24 = 30$

$\Rightarrow$ L.H.S $\neq$ R.H.S

(11) $(3-4)! = 3! - 4!$

Salution. LH.S $(3-4)! = (-1)!$ nat defined.

RH.S $3! - 4! = 6-24 = -18$ Ay.

$\Rightarrow$ LH.S $\neq$ R.H.S

(111) $(2\times3)! = 2!\times3!$

LH.S $(2\times3)! = 6! = 720$

R.H.S $2!\times3! = 2\times1\times3\times2\times1 = 12$

LH.S $\neq$ R.H.S

Ex→7 find x if $\frac{1}{5!} + \frac{1}{6!} = \frac{x}{7!}$

alution. $\frac{1}{5!} + \frac{1}{6!} = \frac{x}{7!}$

$$\frac{1}{5!} + \frac{1}{6\times5!} = \frac{x}{7\times6\times5!}$$

$$\frac{1}{5!}\left(1+\frac{1}{6}\right) = \frac{x}{7\times6\times5!} \Rightarrow \frac{1}{\cancel{5!}}\left(\frac{7}{\cancel{6}}\right)\times7\times\cancel{6}\times\cancel{5!} = x$$

$\Rightarrow 49 = x$ Ay.

Ex→8 If $(n+1)! = 12(n-1)!$ find n.

Solution.

$$(n+1)! = 12(n-1)!$$

$$(n+1)\,n\,\cancel{(n-1)!} = 12\cancel{(n-1)!}$$

$$n^2+n-12 = 0$$

$$(n+4)(n-3) = 0$$

$$n \neq -4,\ n = 3 \text{ Ans}$$

H.W.

Evaluate (i) $\frac{5!-3!}{2!}$ (ii) $\frac{6!}{4!}$ (iii) $\frac{5!}{3!}$

(iv) $\frac{9!}{4! \times 5!}$ (v) $\frac{30!}{28!}$ (vi) $(3!)(4!)$

Q:2 Find x, if $\frac{1}{4!} + \frac{1}{5!} = \frac{x}{6!}$

Q.3 Find n, if $(n+1)! = 30(n-1)!$

Formula (I) $${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

(II) $${}^{n}C_{r} = \frac{n!}{(n-r)! \times r!}$$

Q.1 Evaluate (I) ${}^{7}P_{2}$ (II) ${}^{8}P_{3}$ (III) ${}^{6}P_{3}$

Solution (I) $${}^{7}P_{2} = \frac{7!}{(7-2)!}$$

$$= \frac{7!}{5!} = \frac{7 \times 6 \times \cancel{5!}}{\cancel{5!}} = 42$$

(II) $${}^{8}P_{3} = \frac{8!}{(8-3)!} = \frac{8!}{5!}$$

$$= \frac{8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!}} = 336$$

(III) $${}^{6}P_{3} = \frac{6!}{3!} = \frac{6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}} = 120$$

(IV) $${}^{5}P_{2} = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!}} = 20$$

Q.2 Evaluate ① ${}^{7}C_{3}$ (ii) ${}^{5}C_{2}$ (iii) ${}^{5}C_{3}$ (iv) ${}^{8}C_{2}$

Solution.

① $${}^{7}C_{3} = \frac{7!}{(7-3)! \times 3!} = \frac{7!}{4! \times 3!}$$

$$= \frac{7 \times \cancel{6} \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{3} \times \cancel{2} \times 1} = 35 \text{ Ans}$$

(ii) $${}^{5}C_{2} = \frac{5!}{(5-2)! \times 2!} = \frac{5!}{3! \times 2!}$$

$$= \frac{5 \times \overset{2}{\cancel{4}} \times \cancel{3!}}{\cancel{3!} \times \cancel{2} \times 1} = 10 \text{ Ans}$$

(iii) $${}^{5}C_{3} = \frac{5!}{(5-3)! \times 3!}$$

$$= \frac{5!}{2! \times 3!} = \frac{5 \times \overset{2}{\cancel{4}} \times \cancel{3!}}{\cancel{2} \times 1 \times \cancel{3!}} = 10 \text{ Ans}$$

(iv) $${}^{8}C_{2} = \frac{8!}{(8-2)! \times 2!}$$

$$= \frac{8!}{6! \times 2!} = \frac{\overset{4}{\cancel{8}} \times 7 \times \cancel{6!}}{\cancel{6!} \times \cancel{2} \times 1} = 32 \text{ Ans}$$

Here we note that ${}^{5}C_{2} = {}^{5}C_{3}$

Formula ${}^{n}C_{r} = {}^{n}C_{n-r}$

Q.3 If ${}^{n}P_{r} = 720$, ${}^{n}C_{r} = 120$

Find n and r.

Solution. $${}^{n}P_{r} = 720$$

$$\frac{n!}{(n-r)!} = 720 \quad \text{———} \; ①$$

$${}^{n}C_{r} = 120$$

$$\frac{n!}{(n-r)! \times r!} = 120 \quad \text{———} \; ②$$

Divide ① by ②

$$\frac{\frac{n!}{(n-r)!}}{\frac{n!}{(n-r)! \times r!}} = \frac{720}{120}$$

$$\frac{\cancel{n!}}{\cancel{(n-r)!}} \times \frac{\cancel{(n-r)!} \times r!}{\cancel{(n!)}} = 6$$

$$r! = 6$$

$$r! = 3!$$

$$r = 3$$

Put $r = 3$ in ①

$${}^{n}P_{3} = 720 \Rightarrow \frac{n!}{(n-3)!} = 720$$

Permutation and combination.

Permutation means arrangement

Q.1. In how many ways we can arrange the letter af the ward CAT ?

Salution. CAT
CTA
ATC
ACT
TAC
TCA

Total 6 ways.

If we apply $^{n}P_{r} = \frac{n!}{(n-r)!}$

$$^{n}P_{r} = {}^{3}P_{3} = \frac{3!}{(3-3)!} = \frac{3!}{0!}$$

$$= \frac{3\times2\times1}{1} = 6 \text{ Ans.}$$

Q.2 In how many ways we can arrang the letters af wards FROG?

Salution. FROG,

$$^{4}P_{4} = \frac{4!}{0!} = 24 \text{ Ans.}$$

Q.3. In how manye ways, we can arrang two letter af the ward ANGLE?

Salution. $^5P_2 = \frac{5!}{(5-2)!} = \frac{5!}{3!}$

$= \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!}} = 20$

Q.4 How many words can be farmed from the letters af the word 'TRIANGLE'. How many af these will begin with T and end with E.

Salution. (1) $^8P_8 = 8! = 40320$

(ii) If, we fix T at first place and E at last place, then we have to arrange only 6 words

so $^6P_6 = \frac{6!}{0!} = 720$ Ans

Combination means selection. or choosen.
we have to select only not arrange.
For e.g From letter of word CAT
we have to select 2 letter from that

then we have

CA , AT , CT

only 3 ways.

by Formula $^{3}C_{2} = \frac{3!}{(3-2)!\times 2!}$

$$^{n}C_{r} = \frac{n!}{(n-r)!\times r!}$$

$$^{3}C_{2} = \frac{3!}{1!\times 2!}$$

$$= \frac{3\times\cancel{2}\times 1}{1\times\cancel{2}\times\cancel{1}} = 3 \text{ Ans.}$$

Q.5. From a class of 32 students, 4 are to be chosen for a competition. In how many ways can this be done?

Solution. $^{32}C_{4} = \frac{32!}{28!\times 4!}$

$$= \frac{32\times31\times30\times29\times\cancel{28!}}{\cancel{28!}\times4\times3\times2\times1} = 35960 \text{ Ans}$$

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Chapter - 4

Binomial Thearem

Farmula of binomial thearam for positive index :-

$$(x+y)^n = {}^nC_0 \cdot x^n \cdot y^0 + {}^nC_1 \cdot x^{n-1} \cdot y^1 + {}^nC_2 \cdot x^{n-2} \cdot y^2 + {}^nC_3 \cdot x^{n-3} \cdot y^3 + \_\_\_\_\_\_ + {}^nC_n \cdot x^0 \cdot y^n$$

$(x+y)^n$ total no. of team = n+1

$$\begin{bmatrix} {}^nC_0 = {}^nC_n = 1 \\ {}^nC_1 = n \end{bmatrix} \quad \begin{bmatrix} x^0 = 1 \\ y^0 = 1 \end{bmatrix} \longrightarrow \begin{matrix} {}^3C_0 = 1 \\ {}^3C_3 = 1 \\ {}^3C_1 = 3 \end{matrix}$$

Expand $(x+y)^3$ by 4 sing binomial thearem.

$$(x+y)^3 = {}^3C_0 \cdot x^3 \cdot y^0 + {}^3C_1 \cdot x^2 \cdot y^1 + {}^3C_2 \cdot x^1 \cdot y^2 + {}^3C_3 \cdot x^0 \cdot y^3$$

$$(x+y)^3 = 1 \cdot x^3 \cdot 1 + 3 \cdot x^2 \cdot y + 3 \cdot x^1 y^2 + 1 \cdot 1 \cdot y^3$$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$ Ans

Expand $\left(-2x - \frac{1}{2x}\right)^4$ by 4 sing binomial theorem.

$$\left(-2x - \frac{1}{2x}\right)^4 = {}^4C_0 \cdot (-2x)^4 \cdot \left(\frac{-1}{2x}\right)^0 + {}^4C_1 \cdot (-2x)^3 \cdot \left(\frac{-1}{2x}\right)^1 + {}^4C_2 \cdot (-2x)^2 \cdot \left(\frac{-1}{2x}\right)^2 + {}^4C_3 \cdot (-2x)^1 \cdot \left(\frac{-1}{2x}\right)^3 + {}^4C_4 \cdot (-2x)^0 \cdot \left(\frac{-1}{2x}\right)^4$$

$$\left(-2x - \frac{1}{2x}\right)^4 = 1 \cdot (16x^4) + 4(-8x^3) \cdot \left(\frac{-1}{2x}\right) + 6 \cdot (-4x^2) \cdot \left(\frac{-1}{4x^2}\right) + 9 + 4(-2x) \cdot \left(\frac{-1}{8x^3}\right) + 1 \cdot 1 \cdot \frac{1}{16x^4}$$

$$= 16x^4 + 16x^2 + 6 + \frac{1}{x^2} + \frac{1}{16x^4}$$

Exercise

Q1 How many terms are there in expansion of :-

(a) $(a+3b)^4 = 4+1 = 5$

(b) $[(x-5y)^5]^3 = 3\times5+1 = 15+1 = 16$

(c) $\left(\frac{2}{x}+\frac{x}{2}\right)^8 = 8+1 = 9$

(d) $(x^2+3y^2)^6 = 6+1 = 7$

Q4 Expand the fallowing by using binomial thearem:-

(a) $(2a+3b)^4$

$$(2a+3b)^4 = {}^4C_0 (2a)^4 \cdot (3b)^0 + {}^4C_1 \cdot (2a)^3 (3b)^1 + {}^4C_2 \cdot (2a)^2 \cdot (3b)^2 + {}^4C_3 (2a)^1 (3b)^3 + {}^4C_4 \cdot (2a)^0 \cdot (3b)^4$$

$$= 1 \cdot (16a^4) \cdot 1 + 4 \cdot (8a^3) \cdot (3b) + 6 \cdot (4a^2) \cdot (9b^2) + 4(2a)(27b^3) + 1 \cdot 1 \cdot (81b^4)$$

$$= 16a^4 + 96a^3b + 216a^2b^2 + 216ab^3 + 81b^4$$

(b) $(1-x^2)^4$

$$(1-x^2)^4 = {}^4C_0 \cdot (1)^4 \cdot (-x^2)^0 + {}^4C_1 \cdot (1)^3 \cdot (-x^2)^1 + {}^4C_2 \cdot (1)^2 \cdot (-x^2)^2 + {}^4C_3 \cdot (1)^1 \cdot (-x^2)^3 + {}^4C_4 \cdot (1)^0 \cdot (-x^2)^4$$

$$= 1 \cdot 1 \cdot 1 + 4 \cdot 1(-x^2) + 6 \cdot 1 \cdot (x^4) + 4 \cdot 1 \cdot (-x^6) + (1 \cdot 1 \cdot (x^8))$$

$$= 1 - 4x^2 + 6x^4 - 4x^6 + x^8$$

(c) $(x-2y)^5$

$$(x-2y)^5 = {}^5C_0 (x)^5 (-2y)^0 + {}^5C_1 \cdot (x)^4 \cdot (-2y)^1 + {}^5C_2 (x^3) \cdot (-2y)^2 + {}^5C_3 \cdot (x)^2 \cdot (-2y)^3 + {}^5C_4 \cdot (x^1) \cdot (-2y)^4 + \cancel{{}^5C_5 (x)^0 (-8y^3)}$$

$$\boxed{\cancel{+ {}^5C_5 \cdot (x^0) \cdot (2y)^5}} + {}^5C_5 \cdot (x)^0 \cdot (-2y)^5$$

$$= 1 \cdot (x^5) \cdot 1 + 5 \cdot (x^4) \cdot (-2y) + 10 \cdot (x^3) \cdot (-4y^2) + 10 \cdot (x^2)(-8y^3) + 5 \cdot (x) \cdot (-16y^4) + 1 \cdot 1 (-32y^5)$$

$$= x^5 - 10x^4y - 40x^3y^2 - 80x^3y^3 - 80xy^4 - 32y^5$$

Q1 $(3x+y)^4 = {}^4C_0 (3x)^4 \cdot (y)^0 + {}^4C_1 (3x)^3 \cdot (y)^1 + {}^4C_2 (3x)^2 \cdot (y)^2 + {}^4C_3 \cdot (3x) \cdot (y)^3 + {}^4C_4 (3x)^0 (y)^4$

$$(3x+y)^4 = 1(81x^4) \cdot 1 + 4(27x^3) \cdot (y) + 6(9x^2)(y^2) + 4(1x)(y)^3 + 1(1) \cdot (y^4)$$

$$(3x+y)^4 = 81x^4 + 108x^3y + 54x^2y^2 + 12xy^3 + y^4$$

Q2. $\left(x+\frac{1}{2x}\right)^3 = {}^3C_0 (x)^3 \cdot \left(\frac{1}{2x}\right)^0 + {}^3C_1 (x)^2 \cdot \left(\frac{1}{2x}\right)^1 + {}^3C_2 (x)^1 \cdot \left(\frac{1}{2x}\right)^2 + {}^3C_3 (x)^0 \cdot \left(\frac{1}{2x}\right)^3$

$$= 1x^3 + 3(x^2)\left(\frac{1}{2x}\right) + 3(x) \cdot \left(\frac{1}{4x^2}\right) + 1\left(\frac{1}{8x^3}\right)$$

$$= 1x^3 + \frac{3x}{2} + \frac{3}{4x} + \frac{1}{8x^3}$$

Q3. $(2x-3y)^3 = {}^3C_0 (2x)^3 \cdot (-3y)^0 + {}^3C_1 (2x)^2 \cdot (-3y)^1 + {}^3C_2 (2x)^1 \cdot (-3y)^2 + {}^3C_3 (2x)^0 \cdot (-3y)^3$

$$(2x-3y)^3 = 1(8x^3) + 3(4x^2) \cdot (-3y) + 3(2x) \cdot (9y^2) + 1(-27y^3)$$

$$(2x-3y)^3 = 8x^3 - 36x^2y + 54y^2 - 27y^3$$

Q4 $(99)^3 = (100-1)^3$

$$(99)^3 = {}^3C_0 (100)^3 \cdot (-1)^0 + {}^3C_1 (100)^2 \cdot (-1)^1 + {}^3C_2 (100)^1 \cdot (-1)^2 + {}^3C_3 (100)^0 \cdot (-1)^3$$

$$(99)^3 = 1 \cdot 1000000 \cdot 1 + 3 \cdot 10000 \cdot -1 + 3 \cdot 100 \cdot 1 + 1 \cdot 1 \cdot -1$$

$$(99)^3 = 1000000 - 30000 + 300 - 1$$

$$(99)^3 = 970000 + 299 = 970299$$

Formula of General term in $(x+y)^n$

$$T_{r+1} = {}^nC_r \cdot x^{n-r} \cdot y^r$$

Imp① Find 6th term in the expansion of $(3x+2y)^9$

Solution:- The given expansion is $(3x+2y)^9$

Here, $x = 3x$, $y = 2y$, $n = 9$

$r+1 = 6 \Rightarrow r = 6-1 = 5$

Formula of General term

$$T_{r+1} = {}^nC_r \cdot x^{n-r} \cdot y^r$$

$$T_6 = {}^9C_5 \cdot (3x)^{9-5} \cdot (2y)^5$$

$$T_6 = \frac{9!}{4! \times 5!} \cdot (3x)^4 \cdot (2y)^5$$

$$T_6 = \frac{9\times\overset{2}{\cancel{8}}\times7\times\overset{2}{\cancel{6}}\times\cancel{5!}}{\cancel{4}\times\cancel{3}\times\cancel{2}\times1\times\cancel{5!}} \cdot 81x^4 \cdot 32y^5$$

$$T_6 = 126 \cdot 81x^4 \cdot 32y^5$$

$$T_6 = 326592\, x^4y^5$$

Imp2. Find 4th term in the expansion of $\left(x^2+\frac{1}{2x}\right)^{11}$

Solution:- The given expansion is $\left(x^2+\frac{1}{2x}\right)^{11}$

Here, $x = x^2$, $y = \frac{1}{2x}$, $n = 11$

$r+1 = 4 \Rightarrow r = 4-1 = 3$

Formula of General term

$$T_{r+1} = {}^nC_r \cdot x^{n-r} \cdot y^r$$

$$T_4 = {}^{11}C_3 \cdot (x^2)^{11-3} \cdot \left(\frac{1}{2x}\right)^3$$

$$T_4 = \frac{11!}{8! \times 3!} \cdot (x^2)8 \cdot \frac{1}{8x^3}$$

$$T_4 = \frac{11\times\overset{5}{\cancel{10}}\times\overset{3}{\cancel{9}}\times\cancel{8!}}{\cancel{8!}\times\cancel{3}\times\cancel{2}\times1} \cdot x^{\cancel{16}} \cdot \frac{1}{8x^{\cancel{3}}}$$

$$T_4 = \frac{165x^{13}}{8}$$

Middle term in the expansion of $(x+y)^n$

Case I if n is even then
middle term $\Rightarrow \left(\frac{n}{2}+1\right)$th term.

Case II if n is odd, then
middle term $\Rightarrow \left(\frac{n+1}{2}\right)$th term

$\Rightarrow \left(\frac{n+3}{2}\right)$th term

Q1 Find Middle term in the expansion of $\left(x+\frac{1}{3x}\right)^8$

Solution:- $n = 8$, $x \Rightarrow x$, $y = \frac{1}{3x}$

middle term $= \left(\frac{n}{2}+1\right) = \left(\frac{\cancel{8}}{\cancel{2}}+1\right) = 4+1 = 5$th term

Here $r+1 = 5$, $r = 4$

$$T_{r+1} = {}^nC_r \cdot x^{n-r} \cdot y^r$$

$$T_5 = {}^8C_4 \cdot (x)^{8-4} \cdot \left(\frac{1}{3x}\right)^4$$

$$= \frac{8!}{4! \times 4!} \cdot \cancel{x^4} \cdot \frac{1}{81\cancel{x^4}}$$

$$= \frac{{}^2\cancel{8}\times7\times\overset{2}{\cancel{6}}\times5\times\cancel{4!}}{\cancel{4}\times\cancel{3}\times\cancel{2}\times1\times\cancel{4!}} \cdot \frac{1}{81} = \frac{70}{81}$$

Q9. Find the middle term in the expansion of following:

(i) $\left(x+\frac{1}{2x}\right)^8$

Solution:- $n = 8$, $x = x$, $y = \frac{1}{2x}$

middle term $= \left(\frac{n}{2}+1\right) = \frac{\cancel{8}}{\cancel{2}}+1 = 4+1 = 5$ th term

Here $r+1=5$, $r=5-1=4$

$T_{r+1} = {}^{n}C_{r} \cdot x^{n-r} \cdot y^{r}$

$T_5 = {}^{8}C_4 \cdot x^{8-4} \cdot \left(\frac{1}{2x}\right)^{4}$

$= \frac{8!}{4! \times 4!} \cdot x^{4} \cdot \frac{1}{16x^{4}}$

$= \frac{\cancel{8} \times 7 \times \cancel{6} \times 5 \times \cancel{4!}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{4!}} \cdot \frac{1}{16} = \frac{\cancel{70}^{35}}{\cancel{16}_{8}} = \frac{35}{8}$

(ii) $\left(2x - \frac{x^2}{4}\right)^9$

Solution :- $n=9$, $x=2x$, $y=\frac{-x^2}{4}$

middle term $= \left(\frac{n+1}{2}\right), \left(\frac{n+3}{2}\right)$ th

$= \frac{9+1}{2}, \frac{9+3}{2}$

$= \frac{\cancel{10}}{\cancel{2}} = 5$ th, $\frac{\cancel{12}}{\cancel{2}} = 6$ th

We have to find T_5, T_6

fist we find T_5

$r+1=5$, $r=5-1=4$

$T_{r+1} = {}^{n}C_{r} \cdot x^{n-r} \cdot y^{r}$

$T_5 = {}^{9}C_4 \cdot (2x)^{9-4} \cdot \left(\frac{-x^2}{4}\right)^{4}$

$T_5 = \frac{9!}{5! \times 4!} \cdot 32x^{5} \cdot \frac{-x^{8}}{256}$

$T_5 = \frac{{}^{3}\cancel{9} \times \cancel{8} \times 7 \times \cancel{6} \times \cancel{5!}}{\cancel{5!} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \cdot \cancel{32}x^{5} \cdot \frac{-x^{8}}{\cancel{256}} = \boxed{\frac{\cancel{4032}x^{13}}{\cancel{256}} = \frac{504x^{-13}}{37}}$

1008 504

148

74

37

128

16 4

$T_5 = \frac{63x^{13}}{4}$

Now, we find T_6

$r+1=6$, $r=6-1=5$

Now, $T_{r+1} = {}^{n}C_{r} \cdot x^{n-r} \cdot y^{r}$

$T_6 = {}^{9}C_5 \cdot (2x)^{9-5} \cdot \left(\frac{-x^2}{4}\right)^{5}$

$T_6 = \frac{9!}{4! \times 5!} \cdot 16x^{4} \cdot \frac{-x^{10}}{1024}$

$T_6 = \frac{9 \times \cancel{8} \times 7 \times \cancel{6} \times \cancel{5!}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{5!}} \cdot \frac{\cancel{16}x^{-14}}{\cancel{1024}}$

$T_6 = \boxed{\frac{\cancel{2016}x^{-14}}{\cancel{1024}}} = \frac{-63x^{14}}{32}$

(iii) $\left(x - \frac{1}{x}\right)^{11}$

Solution :- $n=11$, $x=x$, $y=\frac{-1}{x}$

middle term $= \left(\frac{n+1}{2}\right), \left(\frac{n+3}{2}\right)$

$= \frac{11+1}{2}, \frac{11+3}{2}$

$= \frac{\cancel{12}}{\cancel{2}}, \frac{\cancel{14}}{\cancel{2}}$

$= 6$ th, 7 th term

We have to find T_6, T_7

fist we find T_6

$r+1=6$, $r=6-1=5$

$T_{r+1} = {}^{11}C_5 \cdot (x)^{11-5} \cdot \left(\frac{-1}{x}\right)^{5}$

$T_6 = \frac{11!}{6! \times 5!} \cdot x^{6} \cdot \frac{-1}{x^{5}}$

$T_6 = \frac{11 \times \cancel{10} \times \cancel{9} \times \cancel{8} \times 7 \times \cancel{6!}}{\cancel{6!} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \cdot -1x$

$T_6 = -462x$

Now, we find T_7

$r+1=7$, $r=7-1=6$

$T_{r+1} = {}^{11}C_6 \cdot (x)^{11-6} \cdot \left(\frac{-1}{x}\right)^{6}$

$T_7 = \frac{11!}{5! \times 6!} \cdot x^{5} \cdot \frac{-1}{x^{6}}$

$T_7 = \frac{11 \times \cancel{10} \times \cancel{9} \times \cancel{8} \times 7 \times \cancel{6!}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{6!}} \cdot \frac{-1}{x}$

$T_7 = \frac{-462}{x}$

Q2 find 5th term in the expansion of $\left(2x^2 - \frac{1}{3x}\right)^{11}$

Solution :- The given expansion is $\left(2x^2 - \frac{1}{3x}\right)^{11}$

Here, $x = 2x^2$, $y = \frac{-1}{3x}$, $n = 11$

$r+1 = 5 \Rightarrow r = 5-1 = 4$

Formula of General term.

$T_{r+1} = {}^nC_r \cdot x^{n-r} \cdot y^r$

$T_5 = {}^{11}C_4 \cdot (2x^2)^{11-4} \cdot \left(\frac{-1}{3x}\right)^4$

$T_5 = \frac{11!}{8! \times 4!} \cdot (2x^2)^7 \cdot \frac{-1}{81x^4}$

$= \frac{11 \times \overset{5}{\cancel{10}} \times \overset{3}{\cancel{9}} \times \overset{2}{\cancel{8}} \times \cancel{7!}}{\cancel{7!} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \cdot 128x^{\cancel{14}} \cdot \frac{-1}{81\cancel{x^4}}$

$= \frac{-42240x^{10}}{81}$

Q3 find 7th term in the expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$

Solution :- The given expansion is $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$

Here, $x = \frac{4x}{5}$, $y = \frac{-5}{2x}$, $n = 9$

$r+1 = 7$, $r = 7-1 = 6$

Formula of General term

$T_{r+1} = {}^nC_r \cdot x^{n-r} \cdot y^r$

$T_7 = {}^9C_6 \cdot \left(\frac{4x}{5}\right)^{9-6} \cdot \left(\frac{-5}{2x}\right)^6$

$T_7 = \frac{9!}{3! \times 6!} \cdot \frac{64x^3}{125} \cdot \frac{15625}{64x^6}$

$T_7 = \frac{\overset{3}{\cancel{9}} \times \overset{4}{\cancel{8}} \times 7 \times \cancel{6!}}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{6!}} \cdot \frac{\cancel{64x^3}}{\underset{25}{\cancel{125}}} \cdot \frac{\overset{3125}{\cancel{15625}}}{\cancel{64x^6}}$ 125

$T_7 = 84 \cdot \frac{125}{x^3}$

$T_7 = \frac{10500}{x^3}$

Q5 find the 4th term from the end in the expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$

Solution :- 4th term from end $(9-4+2) = 7$th from the beginning

We have to find out T_7

$r+1 = 7$, $r = 7-1 = 6$

$n = 9$, $x = \frac{4x}{5}$, $y \Rightarrow \frac{-5}{2x}$

$T_{r+1} = {}^nC_r \cdot x^{n-r} \cdot y^r$

$T_7 = {}^9C_6 \cdot \left(\frac{4x}{5}\right)^{9-6} \cdot \left(\frac{-5}{2x}\right)^6$

$T_7 = {}^9C_6 \cdot \left(\frac{4x}{5}\right)^3 \cdot \left(\frac{-5}{2x}\right)^6$

$T_7 = \frac{9!}{3! \times 6!} \cdot \frac{\cancel{64x^3}}{\cancel{125}} \cdot \frac{\overset{125}{\cancel{15625}}}{\cancel{64x^6}}$

$T_7 = \frac{\overset{3}{\cancel{9}} \times \overset{4}{\cancel{8}} \times 7 \times \cancel{6!}}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{6!}} \cdot \frac{125}{x^3}$

$T_7 = 84 \cdot \frac{125}{x^3} = \frac{10500}{x^3}$

Q6 find the 5th term from the end in expansion of $\left(x - \frac{1}{x}\right)^{12}$

Solution :- 5th term from end $(12-5+2) = 9$th from the beginning

We have to find out T_9

$r+1 = 9$, $r = 9-1 = 8$

$n = 12$, $x = x$, $y = \frac{-1}{x}$

$$T_{r+1} = {}^{n}C_{r} \cdot x^{n-r} \cdot y^{r}$$

$$T_9 = {}^{12}C_8 \cdot (x)^{12-8} \cdot \left(\frac{-1}{x}\right)^8$$

$$T_9 = {}^{12}C_8 \cdot (x)^4 \cdot \left(\frac{-1}{x}\right)^8$$

$$T_9 = \frac{\overset{3}{\cancel{12}} \times 11 + \overset{5}{\cancel{10}} \times 9 \times \cancel{8!}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{8!}} \cdot x^4 \cdot \frac{+1}{x^8}$$

$$T_9 = \frac{+495}{x^4}$$

Q8 Evaluate the following by using binomial expansion : $(1000)^4$

Solution :- $(999+1)^4$

$(999+1)^4 \Rightarrow {}^4C_0 \cdot (999)^4 \cdot (1)^0 + {}^4C_1 \cdot (999)^3 \cdot (1)^1 + {}^4C_2 \cdot (999)^2 (1)^2$
$+ {}^4C_3 \cdot (999)^1 \cdot (1)^3 + {}^4C_4 \cdot (999)^0 \cdot (1)^4$

$\Rightarrow 996{,}005996001 + 4(997002999)$
$+ 6(998001) + 4(999) + 1 \cdot 1$
$\Rightarrow 996005996001 + 3{,}988{,}011{,}996 +$
$5988006 + 3996 + 1$
$= 999\,999{,}999\,999$

Q8 Evaluate the following by using binomial expansion $(999)^4$

Solution :- $(1000-1)^4$

$(1000-1)^4 = {}^4C_0 \cdot (1000)^4 \cdot (-1)^0 + {}^4C_1 \cdot (1000)^3 \cdot (-1)^1 +$
${}^4C_2 \cdot (1000)^2 \cdot (-1)^2 + {}^4C_3 (1000)^1 \cdot (-1)^3 + {}^4C_4 (1000)^0 \cdot (-1)^4$

$= 1(1000)^4 + 4(1000)^3 (-1) + 6(1000)^2 \cdot 1$
$+ 4(1000) \cdot (-1) + 1 \cdot 1 \cdot 1$

$= 1000000000000 + 4(1000\,000\,000) \cdot (-1) + 6(1000\,000) \cdot 1$
$+ 4000(-1) + 1$
$= 1000000000000 - 4000000000 + 6000\cancel{000} - 4000 + 1$
$= 996{,}005{,}996{,}001$

10 Find the term independent of x in the expansion of:

(i) $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$

Solution! Here $n = 9$, $x = \frac{3x^2}{2}$, $y = \frac{-1}{3x}$

Now $T_{r+1} = {}^{n}C_{r} \cdot x^{n-r} \cdot y^{r}$

$$T_{r+1} = {}^{9}C_{r} \cdot \left(\frac{3x^2}{2}\right)^{9-r} \cdot \left(\frac{-1}{3x}\right)^{r}$$

$$T_{r+1} = {}^{9}C_{r} \cdot \left(\frac{3}{2}\right)^{9-r} \cdot (x^2)^{9-r} \cdot \frac{(-1)^r}{(3)^r \cdot (x)^r}$$

$$T_{r+1} = {}^{9}C_{r} \cdot \left(\frac{3}{2}\right)^{9-r} \cdot x^{18-2r} \cdot \frac{(-1)^r}{3^r \cdot x^r}$$

$$= {}^{9}C_{r} \left(\frac{3}{2}\right)^{9-r} \cdot \frac{(-1)^r}{3^r} \cdot x^{18-2r-r}$$

For term independent of x, put $18 - 3r = 0$
$= r = 6$
put $r = 6$ in eq ①

$$T_7 = {}^{9}C_{6} \cdot \left(\frac{3}{2}\right)^{9-6} \cdot \frac{(-1)^6}{3^6}$$

$$= \frac{9!}{3! \times 6!} \cdot \left(\frac{3}{2}\right)^3 \cdot \frac{(-1)^6}{(3)^6}$$

$$= \frac{3\,\cancel{9} \times \overset{4}{\cancel{8}} \times 7 \times \cancel{6!}}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{6!}} \cdot \frac{\cancel{27}}{\cancel{8}_{2}} \cdot \frac{1}{\cancel{729}_{27}}$$

$$= \overset{7}{\cancel{21}} \cdot \frac{1}{\cancel{54}} = \frac{7}{18} \text{ Ans}$$

Ex - 4.2

Q1. farmula of binomial thearam far any index
If n is a rational no, Positive ,negative , integral or fractional no. and x is a real no. Such that $|x| < 1$, then $(1+x)^n$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} . x^2 + \frac{n(n+1)(n+2)}{3!} . x^3 + \text{----}$$

Q1 If $|x|<1$, write the first three terms in the expansion of the following.

(a) $(1-x)^{-2}$

Solution :- $1+(-2)(-x) + \frac{(-2)(-2-1)}{2!} . (-x)^2 + \text{----}$

$$\Rightarrow 1+2x + \frac{(\cancel{-2})(-3)}{\cancel{2}} . (-x)^2 + \text{----}$$

$$\Rightarrow 1+2x+3x^2 + \text{----}$$

(b) $(1+x)^{-3}$

Solution :- $1+(-3)(x) + \frac{(-3)(-3-1)}{2!} . (x)^2 + \text{----}$

$$\Rightarrow 1-3x + \frac{(-3)(\overset{2}{\cancel{-4}})}{\cancel{2}} . x^2 +$$

$$\Rightarrow 1-3x+6x^2 + \text{----}$$

(c) $(1-x)^{3/2}$

Solution :- $1 + \left(\frac{-3}{2}\right)(-x) + \frac{\left(\frac{-3}{2}\right)\left(\frac{-3}{2}-1\right)}{2!} . (x)^2 + \text{----}$

$$\Rightarrow 1 + \frac{3x}{2} + \left(\frac{-3}{2}\right) . \left(\frac{-5}{2}\right) \times \frac{1}{2} \times x^2 + \text{----}$$

$$\Rightarrow 1 + \frac{3x}{2} + \frac{15x^2}{8} + \text{----}$$

Q2. Write the first three terms in the expansion of $(1-2x^3)^{11/2}$ also state the condition on which the expansion is valid.

Solution :- $(1-2x^3)^{11/2}$

$$\Rightarrow 1+\left(\frac{11}{\cancel{2}}\right)(-\cancel{2}x^3)+\frac{\left(\frac{11}{2}\right)\left(\frac{11}{2}-1\right)}{2!}\cdot(2x^3)^2+ \text{——}$$

$$\Rightarrow 1+11x^3+\frac{11}{2}\times\frac{9}{\cancel{2}}\times\frac{1}{\cancel{2}}\times\cancel{4}x^6+ \text{——}$$

$$\Rightarrow 1-11x^3+\frac{99x^6}{2}+ \text{——}$$

expansion is valid $|-2x^3|<1$

$2|x^3|<1$

$|x^3|<\frac{1}{2}$

Q3 If x be numerically so small that its cube and higher powers may be neglected then find the binomial expansion for:-

(a) $(4+3x)^{-2}$

Solution:- $(4)^{-2}\left[1+\frac{3x}{4}\right]^{-2}$

$$\Rightarrow \frac{1}{16}\left[1+(-\cancel{2})\left(\frac{3x}{\cancel{4}_2}\right)+\frac{(-2)(-2-1)}{2!}\cdot\left(\frac{3x}{4}\right)^2+ \text{——}\right.$$

$$\Rightarrow \frac{1}{16}\left[1+\frac{3x}{2}+\frac{(\cancel{-2})(-3)}{\cancel{2}}\times\frac{9x^2}{16}+ \text{——}\right.$$

$$\Rightarrow \frac{1}{16}\left[1-\frac{3x}{2}+\frac{27x^2}{16}+ \text{——}\right.$$

5 MATRICES AND DETERMINANTS

Matrix :

Definition : A system of mn-numbers (real or complex) arranged in the form of an ordered set of m horizontal lines (called rows) and n vertical lines (called columns) is called an $m \times n$ matrix. We write $m \times n$ matrix as :

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

In concept form the above matrix is represented by $A = [a_{ij}]_{m \times n}$. The numbers $a_{11}, a_{12}, \ldots$ etc are known as the elements of the matrix A. The element a_{ij} belongs to i^{th} row and j^{th} column and is called the $(i, j)^{th}$ element of the matrix $A = [a_{ij}]$. Thus, in the element a_{ij} the first subscript i always denotes the number of row and the second subscript j, number of column in which the element occurs.

For example : $A = \begin{bmatrix} 3 & 1 & 4 \\ 6 & 2 & -3 \end{bmatrix}$ is the matrix having 2 rows and 3 columns and so it is a matrix of order 2 × 3 such that $a_{11} = 3$, $a_{12} = 1$, $a_{13} = 4$, $a_{21} = 6$, $a_{22} = 2$, $a_{23} = -3$.

Types of Matrices

1. **Rectangular Matrix :** Any $m \times n$ matrix $(m \neq n)$ is called a rectangular matrix.

   e.g. $\begin{bmatrix} 3 & 1 & 6 \\ 4 & 1 & 3 \end{bmatrix}$ is a rectangular matrix.

2. **Square Matrix :** Any $n \times n$ matrix is called a square matrix of order n. In this case the number of rows = number of columns.

   e.g. $\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$ is a square matrix and $\begin{bmatrix} 3 & 1 & 4 \\ -3 & 2 & 6 \\ 4 & 1 & 3 \end{bmatrix}$ is a matrix of order 3.

3. **Row Matrix :** A matrix having only one row is called a row matrix or a row vector.

   e.g., $A = \begin{bmatrix} 3 & 6 & 1 & 9 \end{bmatrix}$ is a row matrix of order 1 × 4.

4. **Column Matrix :** A matrix having only one column is called a column matrix or column vector.

   e.g. $A = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$ & $B = \begin{bmatrix} 3 \\ 1 \\ 6 \\ 4 \end{bmatrix}$ are column matrices of order 3 × 1 and 4 × 1 respectively.

5. **Diagonal Matrix :** A square matrix $A = [a_{ij}]$ is said to be diagonal matrix if $a_{ij} = 0$ when $i \neq j$. Thus, it is a square matrix in which all the elements except the diagonal elements are zero.

   e.g. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ is a diagonal matrix of order 3.

6. **Scalar Matrix :** A diagonal matrix is said to be scalar matrix if all its diagonal entries are equal.

   Thus, $A = [a_{ij}]_{m \times n}$ is said to be a scalar matrix.

   $$\text{if } \begin{Bmatrix} a_{ij} = 0 \text{ when } i \neq j \\ a_{ij} = K \text{ when } i = j \end{Bmatrix}$$

   e.g. $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ & $\begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ are scalar matrices of order 2 and 3.

7. **Identity Matrix :** A diagonal matrix is said to be an identity matrix if each of its diagonal elements is unity. This is also known as unit matrix.

   Thus $A = [a_{ij}]_{n \times n}$ is called an identity matrix if

   (*i*) $a_{ij} = 0$ when $i \neq j$.

   (*ii*) $a_{ij} = 1$ when $i = j$.

   The identity matrix of order n is usually denoted by I_n or simply by I.

   e.g. $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; etc.

8. **Zero Matrix :** A matrix is said to be a zero matrix if each of its elements is zero. This is also known as **Null Matrix.** The zero matrix of the type $m \times n$ is denoted by $0_{m \times n}$ or simply by 0.

$$\text{e.g. } 0_{2,3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; 0_{2,2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

9. **Upper Triangular Matrix :** A square matrix $A = [a_{ij}]$ is called an upper triangular matrix if $a_{ij} = 0$ for all $i > j$. Thus in an upper triangular matrix, all elements below the main diagonal are zero.

$$e.g., \; A = \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} \text{ is an upper triangular matrix.}$$

10. **Lower Triangular Matrix :** A square matrix $A = [a_{ij}]$ is called a lower triangular matrix if $a_{ij} = 0$ for all $i < j$.

Thus, in a lower triangular matrix, all elements above the main diagonal are zero.

$$e.g. A = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 6 & 0 \\ 3 & 1 & 4 \end{bmatrix} \text{ is a lower triangular matrix.}$$

Equality of Two Matri :

Ex - 1

Q1 If a matrix has 8 elements. What are the possible orders it can have? What if it has 5 elements.

Solution $8 = 4\times2, 2\times4, 1\times8, 8\times1$

$5 = 5\times1, 1\times5$

Q2. Construct a 3x4 matrix whose elements are given by:

a) $a_{ij} = 2i - j$

Solution :- $a_{ij} = 2i - j$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ (a_{41} \alpha & a_{42} \alpha & a_{43} \alpha & a_{44}) \end{bmatrix}$$

$a_{11} = 2(1) - J$
$= 2 - 1 = 1$

$a_{12} = 2(1) - 2$
$= 2 - 2 = 0$

$a_{13} = 2(1) - 3$
$= 2 - 3 = -1$

$a_{14} = 2(1) - 4$
$= 2 - 4 = -2$

$a_{21} = 2(2) - 1$
$= 4 - 1 = 3$

$a_{22} = 2(2) - 2$
$= 4 - 2 = 2$

$a_{23} = 2(2) - 3$
$= 4 - 3 = 1$

$a_{24} = 2(2) - 4$
$= 4 - 4 = 0$

$a_{31} = 2(3) - 1$
$= 6 - 1 = 5$

$a_{32} = 2(3) - 2$
$= 6 - 2 = 4$

$a_{33} = 2(3) - 3$
$= 6 - 3 = 3$

$a_{34} = 2(3) - 4$
$= 6 - 4 = 2$

$$\text{Hence} = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$$

(ii) $a_{ij} = \dfrac{i + 4j}{2}$

Solution :- $a_{11} = \dfrac{1 + 4(1)}{2} = \dfrac{1+4}{2} = \dfrac{5}{2}$

$a_{12} = \frac{1+4(2)}{2} = \frac{9}{2}$ $\quad a_{22} = \frac{2+4(2)}{2} = \frac{2+8}{2} = 5$

$a_{13} = \frac{1+4(3)}{2} = \frac{1+12}{2} = \frac{13}{2}$ $\quad a_{23} = \frac{2+4(3)}{2} = 7$

$a_{14} = \frac{1+4^{2}(4)}{2} = \frac{17}{2}$ $\quad a_{24} = \frac{2+4(4)}{2} = 9$

$a_{21} = \frac{2+4(1)}{2} = 3$

$a_{31} = \frac{3+4(1)}{2} = \frac{7}{2}$ $\quad a_{32} = 3+4(2) = \frac{3+8}{2} = \frac{11}{2}$

$a_{33} = \frac{3+4(3)}{2} = \frac{15}{2}$ $\quad a_{34} = \frac{3+4(4)}{2} = \frac{19}{2}$

Hence, $\begin{bmatrix} 5/2 & 9/2 & 13/2 & 17/2 \\ 3 & 5 & 7 & 9 \\ 7/2 & 11/2 & 15/2 & 19/2 \end{bmatrix}$

(iii) $a_{ij} = \frac{(i-2j)^2}{2}$

Solution: $a_{11} = \frac{[1\overset{2}{-}2(1)]^2}{2} = \frac{(-1)^2}{2} = \frac{1}{2}$, $a_{12} = \frac{[1-2(2)]^2}{2} = \frac{(-3)^2}{2} = \frac{9}{2}$

$a_{13} = \frac{[1-2(3)]^2}{2} = \frac{(-5)^2}{2} = \frac{25}{2}$, $a_{14} = \frac{[1-2(4)]^2}{2} = \frac{(-7)^2}{2} = \frac{49}{2}$

$a_{21} = \frac{[2-2(1)]^2}{2} = 0$, $a_{22} = \frac{[2-2(2)]^2}{2} = \frac{(-2)^2}{2} = \frac{4}{2}$

$a_{23} = \frac{[2-2(3)]^2}{2} = \frac{(4)^2}{2} = 8$, $a_{24} = \frac{[2-2(4)]^2}{2} = \frac{(-6)^2}{2} = 18$

$a_{31} = \frac{[3-2(1)]^2}{2} = \frac{1}{2}$, $a_{32} = \frac{[3-2(2)]^2}{2} = \left(\frac{-1}{2}\right)^2 = \frac{1}{2}$

$a_{33} = \frac{[3-2(3)]^2}{2} = \frac{(-3)^2}{2} = \frac{9}{2}$, $a_{34} = \frac{[3-2(4)]^2}{2} = \frac{(-5)}{2} = \frac{25}{2}$

Hence, $\begin{bmatrix} 1/2 & 9/2 & 25/2 & 49/2 \\ 0 & 2 & 8 & 18 \\ 1/2 & 1/2 & 9/2 & 25/2 \end{bmatrix}$

(iv) $a_{ij} = i \times j$

$a_{11} = 1$ $\quad a_{21} = 2$ $\quad a_{31} = 3$

$a_{12} = 2$ $\quad a_{22} = 4$ $\quad a_{32} = 6$

$a_{13} = 3$ $\quad a_{23} = 6$ $\quad a_{33} = 9$

$a_{14} = 4$ $\quad a_{24} = 8$ $\quad a_{34} = 12$

Hence = $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}$

3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ then find

$2A - B$

Solution : $2A - B$

$= 2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} -1 & 5 & 3 \\ 5 & 6 & 0 \end{bmatrix}$

4. Compute the indicated products :-

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

Solution $\begin{bmatrix} a^2+b^2 & -ab+ba \\ -ba+ab & b^2+a^2 \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$

5 If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, show that $A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

Solution: L.H.S $\quad A \times A = A^2$

$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1+0 & 1+1 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

L.H.S = R.H.S

6) Find x, y, z, w if $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Solution :- Here, $x-y=-1$ --- ① $2x+z=5$ --- ③

$2x-y=0$ --- ② $3z+w=13$ --- ④

~~Sub~~adding in eq ① and ②

$$\begin{array}{r} x-y=-1 \\ 2x-y=0 \\ \hline -x=-1 \\ x=1 \end{array}$$

We but $x=1$ in eq ①

We get = $1-y=-1$

$-y=-1-1$

$-y=-2$, $y=2$

We put $x=1$ in eq 3

$2(1)+z=5$

$2+z=5$

$z=5-2$

$z=3$

, We put $z=3$ in eq ④

$3(3)+w=13$

$9+w=13$

$w=13-9$

$w=4$

7 If $\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$, find x, y, z, w

Solution, Here, $x=3$

$2x+z=4$ -①

$3x-y=2$ -②

$3y-w=7$ -③

We put the value of x in eq①

$\Rightarrow 2(3)+z=4$

$6+z=4$

$z=-2$

We but $x=3$ in eq ②

$\Rightarrow 3(3)-y=2$

$9-y=2$

$-y=2-9$

$\cancel{-}y=\cancel{-}7$

$y=7$

, We put the value of $y=7$ eq ③

$3y-w=7$

$3(7)-w=7$

$21-w=7$

$-w=7-21$

$\cancel{+}w=\cancel{+}14$

$w=14$

8(i) If $A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$, find the matrix C each that A+B+C is a zero matrix.

Solution Here, $\begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 3+C_{11} & -4+C_{12} & 1+C_{13} \\ 3+C_{21} & 0+C_{22} & 1+C_{23} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$3+C_{11}=0$, $-4+C_{12}=0$, $1+C_{13}=0$,

$C_{11}=-3$ $C_{12}=4$ $C_{13}=-1$

$3+C_{21}=0$ $0+C_{22}=0$ $1+C_{23}=0$

$C_{21}=-3$ $C_{22}=0$ $C_{23}=-1$

Hence $C = \begin{bmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{bmatrix}$

(ii) If $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ -1 & 0 \end{bmatrix}$, find the matrix C Such that A+B+C is a zero matrix

Solution Given $\begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \\ C_{31} & C_{32} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 3+C_{11} & 4+C_{12} \\ 3+C_{21} & 3+C_{22} \\ 2+C_{31} & -1+C_{32} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

$3+C_{11}=0$ $4+C_{12}=0$ $3+C_{21}=0$

$C_{11}=-3$ $C_{12}=-4$ $C_{21}=-3$

$3+C_{22}=0$ $2+C_{31}=0$ $-1+C_{32}=0$

$C_{22}=-3$ $C_{31}=-2$ $C_{32}=1$

9 If $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 1 & -8 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 5 & 6 \\ -1 & 0 & 1 \\ 2 & 1 & 2 \end{bmatrix}$,

$C = \begin{bmatrix} -1 & -2 & 1 \\ -2 & 2 & 3 \\ -1 & -2 & 2 \end{bmatrix}$ then find $A-2B+3C$.

olution Given

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 1 & -8 & 1 \end{bmatrix} - 2\begin{bmatrix} 4 & 5 & 6 \\ -1 & 0 & 1 \\ 2 & 1 & 2 \end{bmatrix} + 3\begin{bmatrix} -1 & -2 & 1 \\ -2 & 2 & 3 \\ -1 & -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 1 & -8 & 1 \end{bmatrix} - \begin{bmatrix} +8 & 10 & 12 \\ 2 & 0 & 2 \\ 4 & 2 & 4 \end{bmatrix} + \begin{bmatrix} -3 & -6 & 3 \\ -6 & 6 & 9 \\ -3 & -6 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1-8-3 & 2-10-6 & 3-12+3 \\ -1-2-6 & 0-0+6 & 2-2+9 \\ 1-4-3 & -8-2-6 & 1-4+6 \end{bmatrix}$$

$$\begin{bmatrix} -10 & -14 & -6 \\ & 6 & 9 \\ -6 & -16 & 3 \end{bmatrix}$$

10 If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 4A + 3I = 0$

Solution: $A^2 = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \cancel{4-(+1)+1} \\ \cancel{+1 \quad 4} \end{bmatrix} \begin{bmatrix} 4+1 & -2-2 \\ -2-2 & 1+4 \end{bmatrix}$

$\begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$

$A^2 - 4A + 3I$

$$\begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} - \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

5.2

1 Evaluate the determinants.

(i) $\begin{vmatrix} -2 & 3 \\ 4 & -9 \end{vmatrix} = 18 - 12 = 6$

(ii) $\begin{vmatrix} x-1 & 2 \\ x^2 & x+1 \end{vmatrix} = (x-1)(x+1) - 2x^2$

$x^2 - 1^2 - 2x^2$

$= -x^2 - 1$

3 If $\begin{vmatrix} 3 & 2 \\ x & 4 \end{vmatrix} = 0$ then find the value of 'x'.

Solution :- $\begin{vmatrix} 3 & 2 \\ x & 4 \end{vmatrix} = 0$

$12 + 2x = 0$

$2x = 12$

$x = \frac{12}{2} = 6$

4 (i) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 3(1+6) - (-4)(1+4) + 5(3-2)$

$= 3(7) + 4(5) + 5(1)$

$= 21 + 20 + 5 = 46$

X (ii) $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix} = 2(0-5) - (-1)(0-(-3)) + (-2)(-5-6)$

$= 2(-5) + 1(3) - 2(-11)$

$= -10 + 3 + 22 = 15$

5 $\Delta = \begin{vmatrix} 3 & 4 & 2 \\ 2 & 1 & 3 \\ -1 & 2 & 4 \end{vmatrix} = 3(4-6) - 4(8-(-3)) + 2(4-(-1))$

$= 3(-2) - 4(11) + 2(5)$

$= -6 - 44 + 10 = -40$

6) (i) $x + y = 2$
$x - y = 0$

Solution $D = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1 + 1 = -2$

$D_1 = \begin{vmatrix} 2 & 1 \\ 0 & -1 \end{vmatrix} = -2$

$D_2 = \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = 0 - 2 = -2$

$x = \frac{D_1}{D}, y = \frac{D_2}{D}$

$x = \frac{-2}{-2}, y = \frac{-2}{-2}$

$x = 1, y = 1$

ii) $x + 2y = 3$
$2x - y = 1$

Solution $D = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -1 - 4 = -5$

$D_1 = \begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix} = -3 - 2 = -5$

$D_2 = \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = 1 - 6 = -5$

$x = \frac{D_1}{D}, y = \frac{D_2}{D}$

$x = \frac{\cancel{-5}}{\cancel{-5}}, y = \frac{\cancel{-5}}{\cancel{-5}}$

$x = 1, y = 1$

(iii) $2x + 3y = 5$
$2x + y = 3$

Solution $D = \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} = 2 - 6 = -4$

$D_1 = \begin{vmatrix} 5 & 3 \\ 3 & 1 \end{vmatrix} = 5 - 9 = -4$

$D_2 = \begin{vmatrix} 2 & 5 \\ 2 & 3 \end{vmatrix} = 6 - 10 = -4$

$x = \frac{D_1}{D}, y = \frac{D_2}{D}$

$x = \frac{\cancel{-4}}{\cancel{-4}}, y = \frac{\cancel{-4}}{\cancel{-4}}$

$x = 1, y = 1$

(iv) $2x + y = 1$
$x - y = 2$

$D = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -2 - 1 = -3$

$D_1 = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -1 - 2 = -3$

$D_2 = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 - 1 = 3$

$x = \frac{D_1}{D}, y = \frac{D_2}{D}$

$x = \frac{\cancel{-3}}{\cancel{-3}}, y = \frac{\cancel{-3}}{\cancel{-3}}$

$x = 1, y = 1$

Trigonometry.

Sexagesimal system

1 right angle $= 90^\circ$

Centesimal system

1 right angle $= 100$ grades

Circular system π radian $= 180^\circ$

1 radian $= \frac{180^\circ}{\pi}$

Imp $\boxed{180^\circ = 200g = \pi \text{ radian}}$

Q.1 Find the radian measures correvesponding to the fallowing degree measures

(i) 25° (ii) 30° (iii) 45° (iv) 60°

(v) 150° (vi) 120° (vii) 270°

Solution: π radian $= 180^\circ$

$\frac{\pi}{180}$ radian $= 1^\circ$

(i) $25^\circ = \overset{5}{\cancel{25}} \times \frac{\pi}{\underset{36}{\cancel{180}}} = \frac{5\pi}{36}$

(ii) $30^\circ = \cancel{30} \times \frac{\pi}{\cancel{180}_6} = \frac{\pi}{6}$

(iii) $\cancel{45}^\circ = \cancel{45} \times \frac{\pi}{\cancel{180}_4} = \frac{\pi}{4}$

(iv) $60^\circ = \cancel{60} \times \frac{\pi}{\cancel{180}} = \frac{\pi}{3}$

(V) 150°

$$\cancel{150}^{5} \times \frac{\pi}{\cancel{180}_{6}} = \frac{5\pi}{6}$$

(VI) 120°

$$\cancel{120}^{2} \times \frac{\pi}{\cancel{180}_{3}} = \frac{2\pi}{3}$$

(VII) 270°

$$\cancel{270}^{3} \times \frac{\pi}{\cancel{180}_{2}} = \frac{3\pi}{2}$$

Q 2 Find the radian measures corresponding to the following degree measures

(I) 90° (II) 360° (III) 135°

Q.3 Find the degree measures corresponding to the following radian measures.

(I) $\left(\frac{5\pi}{3}\right)^c$ (II) $\left(\frac{3\pi}{2}\right)^c$ (III) $\left(\frac{3\pi}{4}\right)^c$ (IV) $\left(\frac{\pi}{2}\right)^c$

Salution

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree}$$

(I) $$\frac{5\pi}{\cancel{3}} \times \frac{\cancel{180}^{60}}{\cancel{\pi}} = 300°$$

(II) $$\frac{3\cancel{\pi}}{\cancel{2}} \times \frac{\cancel{180}^{90}}{\cancel{\pi}} = 270°$$

(iii) $\frac{3\cancel{\pi}}{\cancel{4}} \times \frac{\overset{45}{\cancel{180}}}{\cancel{\pi}} = 135^{\circ}$

(iv) $\frac{\cancel{\pi}}{\cancel{2}} \times \frac{\overset{90}{\cancel{180}}}{\cancel{\pi}} = 90^{\circ}$

Q.3 In Which quadrant the fallowing angles lie?

① 750° ⑪ 890° ⑪ 1160°

(IV) −875 (V) −1850°

Salution ① $750^{\circ} = 2 \times 360 + 30^{\circ}$

It lies in Ist quadrant

$$360 \overline{)\,750\,}\; 2 \qquad \begin{array}{r} 750 \\ 720 \\ \hline 30 \\ \hline \end{array}$$

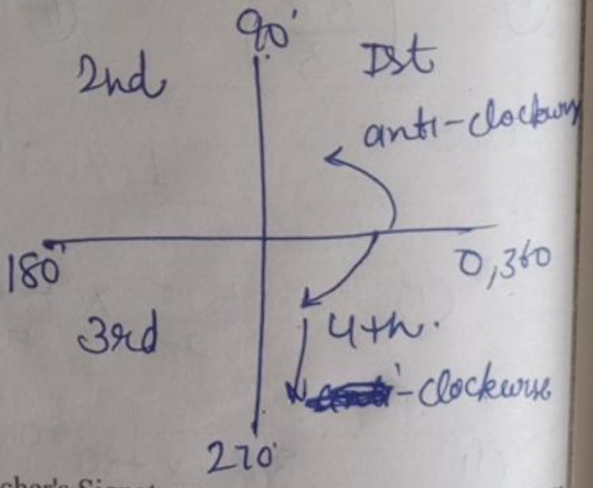
(ii) $890^{\circ} = 2 \times 360 + 170^{\circ}$

It lies in IInd quadrant

$$360 \overline{)\,890\,}\; 2 \qquad \begin{array}{r} 890 \\ 720 \\ \hline 170 \\ \hline \end{array}$$

(III) $1160^{\circ} = 3 \times 360^{\circ} + 80^{\circ}$

It lies in Ist quadrant



(IV) $-875^{\circ} = -2 \times 360 - 155^{\circ}$

It lies in IIIrd quadrant

(V) $-1850^{\circ} = -5 \times 360 - 50^{\circ}$

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It lies in 4th quadrant

Q. 4 Find the degree measure of the fallowing radian measure

(i) $\frac{3\pi}{5}$ (ii) $\frac{2\pi}{3}$ (iii) $\frac{5\pi}{4}$ (iv) $\frac{3\pi}{2}$

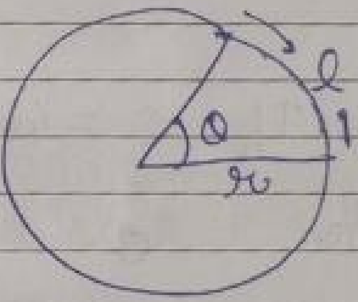
Q. 5 In which quadrant the fallowing angles lie

(i) 480° (ii) 850° (iii) 1550°
(iv) −1150° (v) −1470°

<u>ARc</u> angle Relation

angle of centre $= \frac{\text{arc}}{\text{radius}}$

i.e. $\theta = \frac{l}{r}$



θ is always in radian

Q. 1 In a circle of diameter 40cm, the length of ~~chord~~ arc is 20cm, find the angle subtended at centre

Solution. $d = 40\text{cm}$, radius $= \frac{40}{2} = 20\text{cm}$.
$l = 20\text{cm}$.

$\theta = \frac{l}{r} = \frac{\cancel{20}}{\cancel{20}} = 1$ radian

Q.2, If $l = 11$ cm, $r = 25$ cm.

Find $\theta = ?$

Salution. $\theta = \frac{l}{r} = \frac{11}{25}$ radian.

Q.3 If $\theta = 30^{\circ}$, $r = 5$ cm, find $l = ?$

Salution. $\theta = 30^{\circ} \times \frac{\pi}{180} = \frac{\pi}{6}$

Now, $\theta = \frac{l}{r}$

$$\frac{\pi}{6} = \frac{l}{5} \Rightarrow l = \frac{5\pi}{6}$$

$$= 5 \times \frac{\overset{11}{\cancel{22}}}{7} \times \frac{1}{\underset{3}{\cancel{6}}}$$

$$= \frac{55}{21} \text{ cm.}$$

Q.4 If $\theta = 60^{\circ}$, $l = 10$ cm, find r

Salution. $\theta = 60^{\circ} \times \frac{\pi}{180} = \frac{\pi}{3}$

$l = 10$ cm.

Now $\theta = \frac{l}{r}$

$$\frac{\pi}{3} = \frac{10}{r}$$

$$r = \frac{10 \times 3}{\pi} = \frac{30}{22} \times 7 = \frac{\overset{105}{\cancel{210}}}{\underset{11}{\cancel{22}}} = 9.5 \text{ cm}$$

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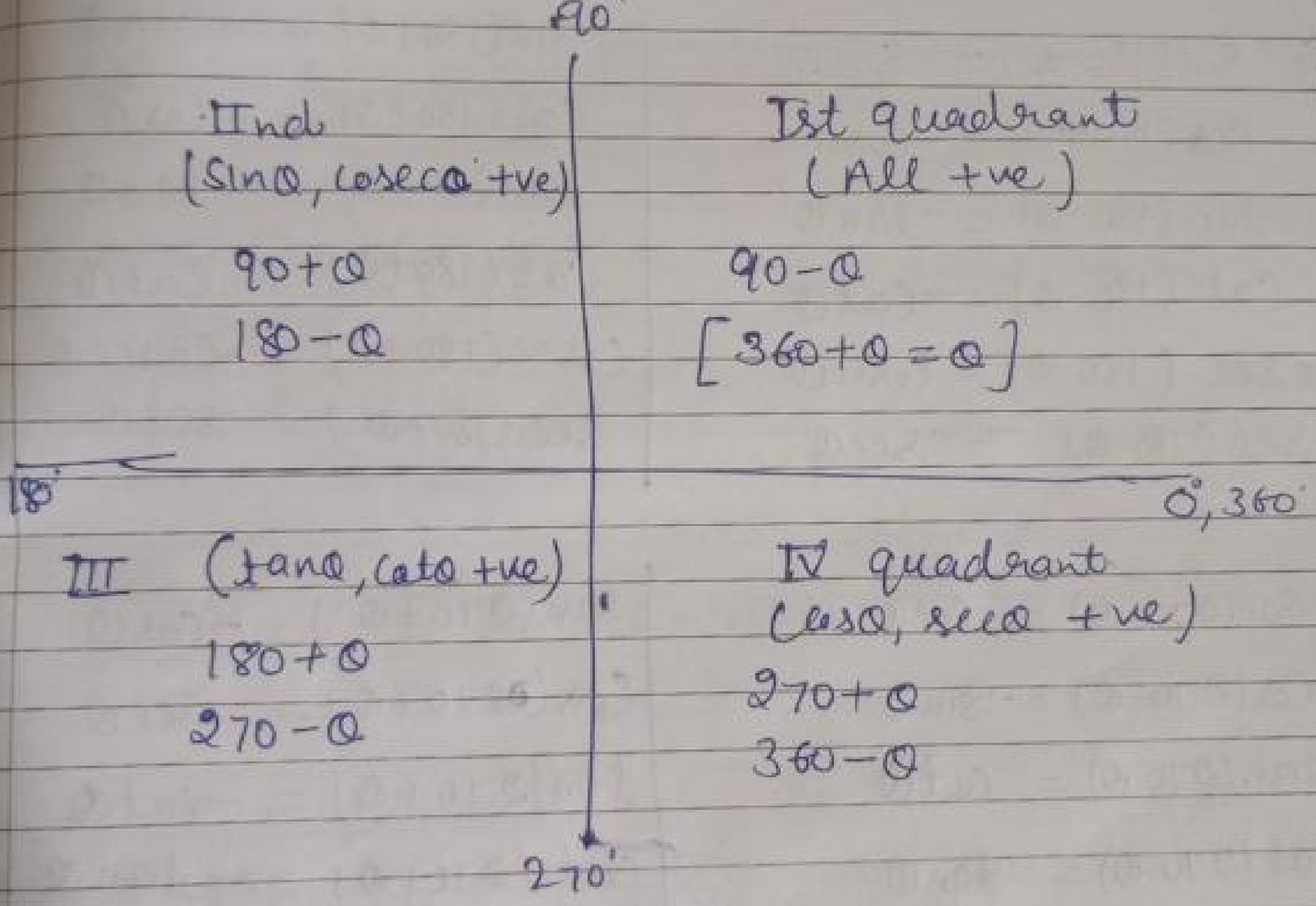
Q.5 If $l = 10\,cm$, $r = 20\,cm$. find θ.

Q.6 If $\theta = 45^\circ$, $r = 7\,cm$. find l.

18/1/22



Add	Sugar	To	Coffee
I	II	III	IV
All	sinθ cosecθ	tanθ cotθ	cosθ secθ

On 90°, 270°
Sin ↔ Cos, tan ↔ Cat, Cosec ↔ Secθ

On 180°, 360°
no change

① $\sin(90-\theta) = \cos\theta$

$\cos(90-\theta) = \sin\theta$

$\tan(90-\theta) = \text{Cat}\,\theta$

$\text{Cat}(90-\theta) = \tan\theta$

$\text{Cosec}(90-\theta) = \sec\theta$

$\sec(90-\theta) = \text{Cosec}\,\theta$

$\sin(90+\theta) = \cos\theta$

$\cos(90+\theta) = -\sin\theta$

$\tan(90+\theta) = -\text{Cat}\,\theta$

$\text{Cat}(90+\theta) = -\tan\theta$

$\text{Cosec}(90+\theta) = \sec\theta$

$\sec(90+\theta) = -\text{Cosec}\,\theta$

② $\sin(180-\theta) = \sin\theta$

$\cos(180+\theta) = -\cos\theta$

$\tan(180-\theta) = -\tan\theta$

$\text{Cat}(180-\theta) = -\text{Cat}\,\theta$

$\text{Cosec}(180-\theta) = \text{Cosec}\,\theta$

$\sec(180-\theta) = -\sec\theta$

$\sin(180+\theta) = -\sin\theta$

$\cos(180+\theta) = -\cos\theta$

$\tan(180+\theta) = \tan\theta$

$\text{Cat}(180+\theta) = \text{Cat}\,\theta$

$\text{Cosec}(180+\theta) = -\text{Cosec}\,\theta$

$\sec(180+\theta) = -\sec\theta$

③ $\sin(270-\theta) = -\cos\theta$

$\cos(270-\theta) = -\sin\theta$

$\tan(270-\theta) = \text{Cat}\,\theta$

$\text{Cat}(270-\theta) = \tan\theta$

$\text{Cosec}(270-\theta) = -\sec\theta$

$\sec(270-\theta) = -\text{Cosec}\,\theta$

$\sin(270+\theta) = -\cos\theta$

$\cos(270+\theta) = \sec\theta$

$\tan(270+\theta) = -\text{Cat}\,\theta$

$\text{Cat}(270+\theta) = -\tan\theta$

$\text{Cosec}(270+\theta) = -\sec\theta$

$\sec(270+\theta) = \text{Cosec}\,\theta$

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$\sin(360-\theta) = -\sin\theta$
$\cos(360-\theta) = \cos\theta$
$\tan(360-\theta) = -\tan\theta$
$\cot(360-\theta) = -\cot\theta$
$\operatorname{cosec}(360-\theta) = -\operatorname{cosec}\theta$
$\sec(360-\theta) = \sec\theta$

$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$
$\tan(-\theta) = -\tan\theta$
$\cot(-\theta) = -\cot\theta$
$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$
$\sec(-\theta) = \sec\theta$

$\sin(360+\theta) = \sin\theta$
$\cos(360+\theta) = \cos\theta$
$\tan(360+\theta) = \tan\theta$
$\cot(360+\theta) = \cot\theta$
$\operatorname{cosec}(360+\theta) = \operatorname{cosec}\theta$
$\sec(360+\theta) = \sec\theta$

Find $\sin 135'$

Salution $\sin 135^\circ = \sin(90+45')$
$= \cos 45' = \frac{1}{\sqrt{2}}$

or $\sin 135' = \sin(180-45')$
$= \sin 45' = \frac{1}{\sqrt{2}}$

② Find $\cos 120'$

Salution $\cos(90+30') = -\sin 30' = -\frac{1}{2}$

$\cos(180-60') = -\cos 60' = \frac{1}{2}$

③ Evaluate tan 225'

Salution. $\tan 225 = \tan(180+45')$

$= \tan 45' = 1$

OR. $\tan 225' = \tan(270'-45') = \cot 45 = 1$

④ Evaluate $\cos 210'$

Salution. $\cos 210' = \cos(180+30')$

$= -\cos 30' = \frac{-\sqrt{3}}{2}$

OR $\cos 210' = \cos(270'-60')$

$= -\sin 60' = \frac{-\sqrt{3}}{2}$

⑤ Evaluate $\sin 300'$

Salution. $\sin 300' = \sin(270+30')$

$= -\cos 30' = -\frac{\sqrt{3}}{2}$

$\sin(360'-60') = -\sin 60' = \frac{\sqrt{3}}{2}$ Ay.

Evaluabe (I) tan135' (II) Cos135' (III) Cos225'

(IV) ~~t~~sec 225' (V) tan210' (VI) Cos300'

Teacher's Signature : ....

Addition and subtraction Formulae

1 $\sin(A+B) = \sin A \cos B + \cos A \sin B$

2 $\sin(A-B) = \sin A \cos B - \cos A \sin B$

3 $\cos(A+B) = \cos A \cos B - \sin A \sin B$

4 $\cos(A-B) = \cos A \cos B + \sin A \sin B$

5 $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

6 $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

7. $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot A + \cot B}$

8. $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

9. $\sin(A+B) \cdot \sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

10. $\cos(A+B) \cdot \cos(A-B) = \cos^2 B - \sin^2 A = \cos^2 A - \sin^2 B$

Q.1 Find the values of following

(1) $\sin 105^\circ = \sin(60^\circ + 45^\circ)$

$= \sin 60^\circ \cos 45 + \cos 60^\circ \sin 45^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$ Ans.

$$\left[\begin{array}{l} \because \sin(A+B) \\ = \sin A\cos B \\ \quad + \cos A \sin B \end{array}\right]$$

② $\sin 75 = \sin(45+30^\circ)$

$= \sin 45 \cos 30^\circ + \cos 45 \sin 30^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

③ $\cos 105^\circ = \cos(60^\circ+45^\circ)$

$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$ Ans.

$$\left[\begin{array}{l} \cos(A+B) \\ = \cos A \cos B \\ \quad - \sin A \sin B \end{array}\right]$$

④ $\cos 75^\circ = \cos(45^\circ+30^\circ)$

$= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$ Ans.

⑤ $\sin 15^\circ = \sin(45-30^\circ)$

$= \sin 45 \cos 30 - \cos 45^\circ \sin 30^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

⑥ $\cos 15^\circ = \cos(45^\circ-30^\circ)$

$= \cos 45 \cos 30 + \sin 45^\circ \sin 30^\circ$

[∵ $\cos(A-B) = \cos A \cos B + \sin A \sin B$]

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$ Ans.

⑦ $\tan 75^\circ = \tan(45^\circ+30)$

$= \frac{\tan 45 + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$

$= \frac{1 + \left(\frac{1}{\sqrt{3}}\right)}{1-(1)\left(\frac{1}{\sqrt{3}}\right)} = \cancel{\frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\sqrt{3}-1}}$

$= \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\cancel{\sqrt{3}}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ Ans

[∵ $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$]

⑧ $\tan 15^\circ = \tan(45^\circ - 30^\circ)$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + (1)\left(\frac{1}{\sqrt{3}}\right)} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3} - 1}{\sqrt{3}}}{\frac{\sqrt{3} + 1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$ Ans.

$\left[\because \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}\right]$

Evaluate

① $\sin 50^\circ \cos 20^\circ - \cos 50^\circ \sin 20^\circ$

$= \sin(50 - 20)$

$= \sin 30^\circ = \frac{1}{2}$

$\left[\because \sin A \cos B - \cos A \sin B = \sin(A - B)\right]$

② $\sin 22 \cos 8 + \cos 22 \sin 8$

$\sin(22 + 8)$

$\sin 30^\circ = \frac{1}{2}$

$\left[\because \sin(A + B) \quad \sin A \cos B + \cos A \sin B\right]$

③ $\cos 50 \cos 10 - \sin 50 \sin 10^\circ$ $\left[\because \cos A \cos B - \sin A \sin B = \cos(A+B)\right]$

$\cos(50 + 10^\circ)$

$= \cos 60^\circ = \frac{1}{2}$

④ $\cos 40^\circ \cos 10^\circ + \sin 40^\circ \sin 10^\circ$ $\left[\because \cos A \cos B + \sin A \sin B = \cos(A-B)\right]$

$\cos(40^\circ - 10^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$

⑤ Prove that $\tan 58 = \frac{\cos 13^\circ + \sin 13^\circ}{\cos 13^\circ - \sin 13^\circ}$

Solution. L.H.S.

C.W

① Evaluate

① $\sin 105$ ② $\cos 75$ ③ $\tan 75^\circ$

④ $\tan 105^\circ$

⑤ $\cos 50 \cos 10 - \sin 50 \sin 10$

⑥ $\cos 40 \cos 20 - \sin 40 \sin 20$

⑦ $\sin 35 \cos 25 + \cos 35 \sin 25$

⑧ $\sin 50 \cos 20 - \cos 50 \sin 20^\circ$

⑨ $\frac{\tan 35 - \tan 5}{1 + \tan 35 \cdot \tan 5}$

⑩ $\frac{\tan 40 + \tan 20}{1 - \tan 40 \cdot \tan 20}$

⑥ $$\frac{\tan 65 - \tan 5}{1 + \tan 65 . \tan 5} = \tan(65-5)$$
$$= \tan 60 = \sqrt{3}$$

$$\left[\frac{\tan A - \tan B}{1 + \tan A . \tan B} = \tan(A-B) \right]$$

⑦ $$\frac{\tan 40 + \tan 20}{1 - \tan 40 . \tan 20} = \tan(40+20) = \tan 60^\circ = \sqrt{3}$$

⑧ Evaluate $\cot 75^\circ$

Solution $\cot 75^\circ = \cot(45+30^\circ)$

$$= \frac{\cot 45 \cot 30 - 1}{\cot 45 + \cot 30}$$

$$= \frac{(1)(\sqrt{3}) - 1}{1 + \sqrt{3}} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \text{ Ans.}$$

⑨ $\cot 15 = \cot(45-30^\circ)$

$$= \frac{\cot 45 \cot 30 + 1}{\cot 30 - \cot 45}$$

$$= \frac{(1)(\sqrt{3}) + 1}{\sqrt{3} - 1} = \frac{\sqrt{3}+1}{\sqrt{3}-1} \text{ Ans.}$$

Q.1 Prove that $\tan 58° = \frac{\cos 13° + \sin 13°}{\cos 13° - \sin 13°}$

Solution. Taking L.H.S

$\tan 58 = \tan(45° + 13)$

$= \frac{\tan 45° + \tan 13}{1 - \tan 45 . \tan 13} = \frac{1 + \tan 13}{1 - (1) . \tan 13}$

$= \frac{1 + \tan 13}{1 - \tan 13}$

$= \frac{1 + \frac{\sin 13}{\cos 13}}{1 - \frac{\sin 13}{\cos 13}} = \frac{\frac{\cos 13 + \sin 13}{\cancel{\cos 13}}}{\frac{\cos 13 - \sin 13}{\cancel{\cos 13}}}$

$= \frac{\cos 13° + \sin 13}{\cos 13 - \sin 13°}$ Ans.

Q.2 Prove that $\tan 33 = \frac{\cos 12 - \sin 12}{\cos 12 + \sin 12}$

Solution.
L.H.S. $\tan 33 = \tan(45 - 12) = \frac{\tan 45 - \tan 12}{1 + \tan 45 . \tan 12}$

$\tan 33 = \frac{1 - \tan 12}{1 + (1) . \tan 12}$

$= \frac{1 - \tan 12}{1 + \tan 12} = \frac{1 - \frac{\sin 12}{\cos 12}}{1 + \frac{\sin 12}{\cos 12}}$

$= \frac{\frac{\cos 12 - \sin 12}{\cancel{\cos 12}}}{\frac{\cos 12 + \sin 12}{\cancel{\cos 12}}} = \frac{\cos 12 - \sin 12}{\cos 12 + \sin 12} = R.H.S$

Q.3 Prove that

$\tan 7A - \tan 4A - \tan 3A = \tan 7A \cdot \tan 4A \cdot \tan 3A$

Solution. Take $7A = 3A + 4A$

$\tan 7A = \tan(3A + 4A)$

$$\frac{\tan 7A}{1} = \frac{\tan 3A + \tan 4A}{1 - \tan 3A \cdot \tan 4A}$$

$\tan 7A(1 - \tan 3A \cdot \tan 4A) = \tan 3A + \tan 4A$

$\tan 7A - \tan 7A \cdot \tan 3A \cdot \tan 4A = \tan 3A + \tan 4A$

$\tan 7A - \tan 3A - \tan 4A = \tan 7A \cdot \tan 3A \cdot \tan 4A.$

L.H.S = R.H.S

Q.4 Prove that

$\tan 11A - \tan 7A - \tan 4A = \tan 11A \tan 7A \tan 4A$

Solution. Take $11A = 7A + 4A$

$\tan 11A = \tan(7A + 4A)$

$$\frac{\tan 11A}{1} = \frac{\tan 7A + \tan 4A}{1 - \tan 7A \cdot \tan 4A}$$

$\tan 11A(1 - \tan 7A \cdot \tan 4A] = \tan 7A + \tan 4A$

$\tan 11A - \tan 11A \cdot \tan 7A \cdot \tan 4A = \tan 7A + \tan 4A$

$\tan 11A - \tan 7A - \tan 4A = \tan 11A \cdot \tan 7A \cdot \tan 4A$

LH.S = RH.S

Teacher's Signature : ....

Q.5 If $\sin A = \frac{4}{5}$, $\cos B = \frac{5}{13}$, $0 < A, B < \frac{\pi}{2}$,
find value of ① $\sin(A+B)$ ⑪ $\cos(A+B)$

Solution.

$$\sin A = \frac{4}{5} \quad , \quad \cos B = \frac{5}{13}$$

We know that $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$\cos A = \sqrt{1-\sin^2 A}$$
$$= \sqrt{1-\left(\frac{4}{5}\right)^2}$$
$$= \sqrt{1-\frac{16}{25}}$$
$$= \sqrt{\frac{25-16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\cos B = \frac{5}{13}$$
$$\sin B = \sqrt{1-\cos^2 B}$$
$$= \sqrt{1-\left(\frac{5}{13}\right)^2}$$
$$= \sqrt{1-\frac{25}{169}}$$
$$= \sqrt{\frac{144}{169}} = \frac{12}{13}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$= \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$$
$$= \frac{20}{65} + \frac{36}{65} = \frac{20+36}{65} = \frac{56}{65} \text{ Ans}$$

Now ⑪ $\cos(A+B) = \cos A \cos B - \sin A \sin B$
$$= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$$
$$= \frac{15}{65} - \frac{48}{65} = \frac{15-48}{65} = \frac{-33}{65}$$

Q.6 If $\sin A = \frac{3}{5}$, $\pi < A < \frac{3\pi}{2}$

and $\cos B = -\frac{12}{13}$, $0 < B < \frac{\pi}{2}$, find $\sin(A-B)$

Solution.

$\sin A = \frac{3}{5}$

$\cos A = -\sqrt{1-\sin^2 A}$ $\left[\pi < A < \frac{3\pi}{2}\right]$

$= -\sqrt{1-\left(\frac{3}{5}\right)^2}$

$= -\sqrt{1-\frac{9}{25}}$

$= -\sqrt{\frac{25-9}{25}}$

$= -\sqrt{\frac{16}{25}} = -\frac{4}{5}$

$\cos B = -\frac{12}{13}$

$\sin B = \sqrt{1-\cos^2 B}$

$= \sqrt{1-\left(-\frac{12}{13}\right)^2}$

$= \sqrt{1-\frac{144}{169}}$

$= \sqrt{\frac{25}{169}} = \frac{5}{13}$

Now

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$= \left(\frac{3}{5}\right)\left(-\frac{12}{13}\right) - \left(-\frac{4}{5}\right)\left(\frac{5}{13}\right)$

$= -\frac{36}{65} + \frac{20}{65} = \frac{-36+20}{65} = -\frac{16}{65}$ Ans

Teacher's Signature : ....

Transfarmation Farmulae

Farmulae to Transfarm the product into sum or difference

~~$\sin(A+B) = \sin A \cos B + C$~~

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B).$$

C-D farmulae

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$$

Q.1 change the fallowing products into sum or difference af t-ratios

① $2\sin 8\theta\cos 5\theta$ (ii) $2\cos 9\theta\sin 4\theta$

Salution

$$2\sin 8\theta\cos 5\theta = \sin(8\theta+5\theta) + \sin(8\theta-5\theta)$$

$$= \sin 13\theta + \sin 3\theta$$

$$\left[2\sin A\cos B = \sin(A+B) + \sin(A-B)\right]$$

(ii) $2\cos 9\theta \sin 4\theta$

$2\cos A \sin B = \sin(A+B) - \sin(A-B)$

$= \sin(9\theta + 4\theta) - \sin(9\theta - 4\theta)$

$= \sin 13\theta - \sin 5\theta$

(iii) $\cos 7\theta \cos 3\theta$

Salution $\because \cos 7\theta \cos 3\theta$

multiplying and divide by 2

$\frac{1}{2}[2\cos 7\theta \cos 3\theta]$

$\frac{1}{2}[\cos(7\theta + 3\theta) + \cos(7\theta - 3\theta)]$

$= \frac{1}{2}[\cos 10\theta + \cos 4\theta]$

(iv) $2\sin 75 \cdot \sin 15$

Salution.

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$2\sin 75 \cdot \sin 15 = \cos(75-15) - \cos(75+15)$

$= \cos 60 - \cos 90'$

$= \frac{1}{2} - 0 = \frac{1}{2}$

Q.2 Prove that $\sin(45' + A)\sin(45 - A) = \frac{1}{2}\cos 2A$

Salution

Take L.H.S

$\sin(45' + A)\sin(45' - A)$

$\frac{1}{2}[2\sin(45+A) \cdot \sin(45' - A)]$

$\left[\begin{matrix} 2\sin A \sin B \\ \cos(A-B) \\ -\cos(A+B) \end{matrix}\right]$

$\frac{1}{2}[\cos[\cancel{45} + A - \cancel{45} + A] - \cos[45' + \cancel{A} + 45 \cancel{-A}]]$

$= \frac{1}{2}\left[\cos 2A - \cos 90^\circ\right]$ $\quad [\because \cos 90 = 0]$

$= \frac{1}{2}\left[\cos 2A\right]$

8 marks question

V.Imp Prove that

$\sin 10^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{8}$

L.H.S. $\sin 10 \sin 50^\circ \sin 70^\circ$

$\frac{1}{2}\left[2\sin 10 \cdot \sin 50\right] \cdot \sin 70^\circ$ $\quad \left[\because 2\sin A \sin B = \cos(A-B) - \cos(A+B)\right]$

$\frac{1}{2}\left[\cos(10-50) - \cos(10+50)\right] \cdot \sin 70^\circ$

$\frac{1}{2}\left[\cos(-40) - \cos 60^\circ\right] \cdot \sin 70^\circ$

$\frac{1}{2}\left[\cos 40 - \frac{1}{2}\right] \cdot \sin 70$ $\quad [\cos(-\theta) = \cos\theta]$

$\frac{1}{2}\left[\cos 40^\circ \sin 70^\circ - \frac{1}{2}\sin 70^\circ\right]$

$\frac{1}{2}\left[\frac{1}{2}(2\cos 40^\circ \sin 70^\circ) - \frac{1}{2}\sin 70^\circ\right]$

~~[2 Cos A Sin B =~~

$\frac{1}{2}\left[\frac{1}{2}(2\sin 70 \cos 40) - \frac{1}{2}\sin 70^\circ\right]$

$[\because 2\sin A \cos B = \sin(A+B) + \sin(A-B)]$

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$\frac{1}{2}\left[\frac{1}{2}\{\sin(70+40) + \sin(70-40)\} - \frac{1}{2}\sin 70\right]$

$\frac{1}{2}\left[\frac{1}{2}(\sin 110 + \sin 30) - \frac{1}{2}\sin 70^\circ\right]$

$\frac{1}{2}\left[\frac{1}{2}\sin(180^\circ - 70) + \left(\frac{1}{2}\right)\frac{1}{2} - \frac{1}{2}\sin 70^\circ\right]$

$\frac{1}{2}\left[\cancel{\frac{1}{2}\sin 70} + \frac{1}{2}\left(\frac{1}{2}\right) - \cancel{\frac{1}{2}\sin 70^\circ}\right]$ $\quad [\because \sin(180-\theta) = \sin\theta]$

$= \frac{1}{2}\left(\frac{1}{4}\right) = \frac{1}{8}$

Prove that $\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$

Salution. L.H.S

$\cos 10 \cdot \cos 30 \cdot \cos 50 \cos 70^\circ$ $\quad [\because 2\cos A \cos B = \cos(A+B) + \cos(A-B)]$

$\frac{\sqrt{3}}{2} \cos 10 \cdot \cos 50 \cdot \cos 70$

$\frac{\sqrt{3}}{4}(2\cos 10 \cos 50^\circ) \cdot \cos 70^\circ$

$\frac{\sqrt{3}}{4}\left[\cos(10+50) + \cos(10-50)\right] \cdot \cos 70$

$\frac{\sqrt{3}}{4}\left[\cos 60 + \cos 40\right] \cdot \cos 70^\circ$

$\frac{\sqrt{3}}{4}\left[\frac{1}{2} + \cos 40\right] \cdot \cos 70^\circ$

$\frac{\sqrt{3}}{4}\left[\frac{1}{2}\cos 70^\circ + \cos 40 \cdot \cos 70^\circ\right]$

$\frac{\sqrt{3}}{4}\left[\frac{1}{2}\cos 70^\circ + \frac{1}{2}(2\cos 40 \cdot \cos 70)\right]$

$\frac{\sqrt{3}}{4}\left[\frac{1}{2}\cos 70^\circ + \frac{1}{2}\left[\cos(40+70) + \cos(40-70^\circ)\right]\right]$

$= \frac{\sqrt{3}}{4}\left[\frac{1}{2}\cos 70 + \frac{1}{2}\left(\cos 110 + \cos 30\right)\right]$

$= \frac{\sqrt{3}}{4}\left[\frac{1}{2}\cos 70 + \frac{1}{2}\cos 110 + \frac{1}{2}\cos 30\right]$

$= \frac{\sqrt{3}}{4}\left[\frac{1}{2}\cos 70 + \frac{1}{2}\cos(180-70) + \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)\right]$

$= \frac{\sqrt{3}}{4}\left[\cancel{\frac{1}{2}\cos 70} - \cancel{\frac{1}{2}\cos 70} + \frac{\sqrt{3}}{4}\right]$

$= \frac{3}{16}$ = R.H.S

$\left[\cos(180-\theta) = -\cos\theta\right]$

CW

Evaluate, OR change the product into sum or difference

(i) $2\sin 7\theta \cos 4\theta$

(ii) $2\cos 8\theta \sin 2\theta$

(iii) $2\cos 75 \cos 15$

(iv) $2\sin 9\theta \sin 5\theta$

Experiment No. Transformation formulae.

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$$

Express each of the following as product

① $\sin 14\theta + \sin 3\theta$ (ii) $\sin 5\theta - \sin 3\theta$

(iii) $\cos 8\theta + \cos 2\theta$ (iv) $\cos 10' - \cos 50$.

Solution. ① $\sin 14\theta + \sin 3\theta = 2\sin\left(\frac{14\theta+3\theta}{2}\right)\cos\left(\frac{14\theta-3\theta}{2}\right)$

$$= 2\sin\left(\frac{17\theta}{2}\right)\cos\left(\frac{11\theta}{2}\right)$$

(ii) $\sin 5\theta - \sin 3\theta$

$$= 2\cos\left(\frac{5\theta+3\theta}{2}\right)\sin\left(\frac{5\theta-3\theta}{2}\right)$$

$$= 2\cos\left(\frac{8\theta}{2}\right)\sin\left(\frac{2\theta}{2}\right)$$

$$= 2\cos 4\theta \sin\theta$$

(iii) $\cos 8\theta + \cos 2\theta$

$$= 2\cos\left(\frac{8\theta+2\theta}{2}\right)\cos\left(\frac{8\theta-2\theta}{2}\right)$$

$$= 2\cos\left(\frac{10\theta}{2}\right)\cdot\cos\left(\frac{6\theta}{2}\right)$$

$$= 2\cos 5\theta\cdot\cos 3\theta$$

(iv) $\cos 10^\circ - \cos 50^\circ$

$\left[\cos C - \cos D = 2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{D-C}{2}\right) \right]$

Solution. $\cos 10 - \cos 50^\circ$

$2 \cancel{\sin} \left(\frac{10+50}{2}\right) \cdot \sin\left(\frac{50-40}{2}\right)$

$2 \sin\left(\frac{60}{2}\right) \sin\left(\frac{40}{2}\right)$

$= 2 \sin 30 \cdot \sin 20 = 2 \times \frac{1}{2} \times \sin 20^\circ$

$= \sin 20^\circ$

Q. 2 Prove that $\frac{\sin A + \sin 3A}{\cos A + \cos 3A} = \tan 2A$

Solution. L.H.S

$\frac{\sin A + \sin 3A}{\cos A + \cos 3A} = \frac{2 \sin\left(\frac{A+3A}{2}\right) \cos\left(\frac{A-3A}{2}\right)}{2 \cos\left(\frac{A+3A}{2}\right) \cos\left(\frac{A-3A}{2}\right)}$

$= \frac{2 \sin\left(\frac{4A}{2}\right) \cdot \cos\left(-\frac{2A}{2}\right)}{2 \cos\left(\frac{4A}{2}\right) \cos\left(-\frac{2A}{2}\right)}$ $\left[\cos(-\theta) = \cos\theta \right]$

$= \frac{\cancel{2} \sin 2A \cdot \cancel{\cos A}}{\cancel{2} \cos 2A \ \cancel{\cos A}} = \tan 2A$

Q. 3 Prove that $\cos 52 + \cos 68 + \cos 172 = 0$

Solution. L.H.S $\cos 52 + \cos 68 + \cos 172$

$2 \cos\left(\frac{52+68}{2}\right) \cos\left(\frac{52-68}{2}\right) + \cos 172$

$2 \cos\left(\frac{120}{2}\right) \cdot \cos\left(\frac{-16}{2}\right) + \cos 172$

$2 \cos(60^\circ) \cdot \cos 8 + \cos 172$

$\cancel{2} \times \frac{1}{\cancel{2}} \cdot \cos 8 + \cos 172$

$\cos 8 + \cos 172$

Again Apply $\cos C + \cos D$

$2 \cos\left(\frac{8+172}{2}\right) \cos\left(\frac{8-172}{2}\right)$

$2 \cos\left(\frac{180}{2}\right) \cdot \cos\left(-\frac{164}{2}\right)$

$2 \cdot \cos 90^\circ \cdot \cos(-81) = 2 \times 0 \times \cos 81 = 0 = $ RHS

Q. 4. Prove that

$\frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A} = \tan 6A$

Solution. L.H.S

$\frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A}$

$\frac{(\sin 3A + \sin 9A) + (\sin 5A + \sin 7A)}{(\cos 3A + \cos 9A) + [\cos 5A + \cos 7A]}$

$\frac{2 \sin\left(\frac{12A}{2}\right) \cos\left(-\frac{6A}{2}\right) + 2 \sin\left(\frac{12A}{2}\right) \cos\left(-\frac{2A}{2}\right)}{2 \cos\left(\frac{12A}{2}\right) \cos\left(-\frac{6A}{2}\right) + 2 \cos\left(\frac{12A}{2}\right) \cos\left(-\frac{2A}{2}\right)}$

$= \frac{2 \sin 6A \cos 3A + 2 \sin 6A \cos A}{2 \cos 6A \cos 3A + 2 \cos 6A \cos A}$

$$= \frac{2\sin 6A\ \cancel{(\cos 3A + \cos A)}}{2\cos 6A\ \cancel{(\cos 3A + \cos A)}} = \tan 6A = RH.S$$

Q. 5 Prove that

$\cos 3\theta + \cos 5\theta + \cos 7\theta + \cos 15\theta = 4\cos 4\theta \cos 5\theta \cos 6\theta$.

Solution. L.HS

$$\cos 3\theta + \cos 5\theta + \cos 7\theta + \cos 15\theta$$

$$\underbrace{\cos 3\theta + \cos 15\theta} + \underbrace{\cos 5\theta + \cos 7\theta}$$

$$2\cos\left(\frac{3\theta + 15\theta}{2}\right)\cdot\cos\left(\frac{3\theta - 15\theta}{2}\right)$$

$$+ 2\cos\left(\frac{5\theta + 7\theta}{2}\right)\cos\left(\frac{5\theta - 7\theta}{2}\right)$$

$$= 2\cos(9\theta)\cos(+6\theta) + 2\cos 6\theta\cos(+\theta)$$

$$= 2\cos 6\theta\,[\cos 9\theta + \cos\theta]$$

$$= 2\cos 6\theta\left[2\cos\left(\frac{9\theta + \theta}{2}\right)\cos\left(\frac{9\theta - \theta}{2}\right)\right]$$

$$= 2\cos 6\theta\left[2\cos\left(\frac{10\theta}{2}\right)\cdot\cos\left(\frac{8\theta}{2}\right)\right]$$

$$= 2\cos 6\theta\,[2\cos 5\theta\cos 4\theta]$$

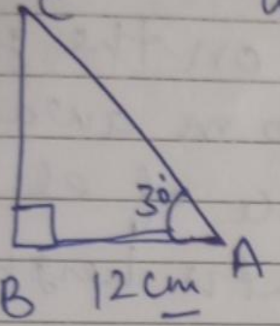
$$= 4\cos 4\theta\cos 5\theta\,.\cos 6\theta = RH.S$$

$[\because \cos(-\theta) = \cos\theta]$

Application of Trigonometry

Q.1 In ΔABC, right angled at B, $AB = 12$ cm and $\angle A = 30^\circ$, find BC and AC

Salution



We have to find out AC, BC

$$\frac{AB}{BC} = \frac{\text{base}}{\text{perpendicular}} = \cot 30^\circ$$

~~$\frac{AB}{BC}$~~ $= \sqrt{3}$

$$\frac{12}{BC} = \sqrt{3} \Rightarrow \frac{12}{\sqrt{3}} = BC$$

Now,

$$\frac{AB}{AC} = \cos 30^\circ$$

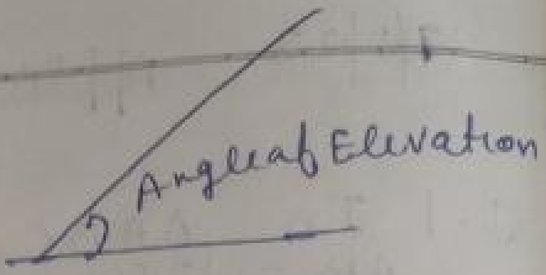
$$\frac{AB}{AC} = \frac{\sqrt{3}}{2} \Rightarrow \frac{12}{AC} = \frac{\sqrt{3}}{2}$$

$$AC = \frac{12 \times 2}{\sqrt{3}} = \frac{24}{\sqrt{3}} \text{ Ans}$$

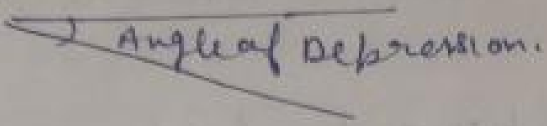
C.W.

If $\angle B = 90^\circ$, $\angle A = 60^\circ$ and $AB = 6$ cm.
find AC and BC

Angle of Elevation.

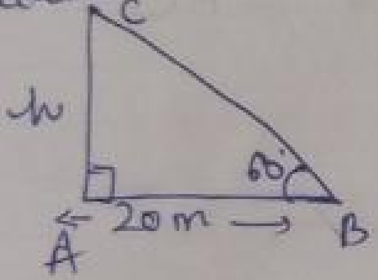


Angle of Depression



Q.1 A tower stand vertically on the ground From a point on the ground 20 m away from the foot of the tower, the angle of elevation of the top of the tower is 60°. What is height of tower?

Solution:



From △ABC, $\frac{AC}{AB} = \tan 60^\circ$

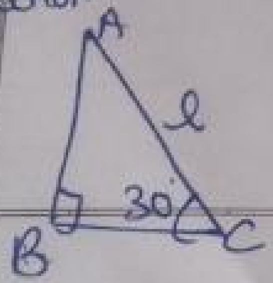
$$\frac{h}{20} = \tan 60^\circ$$

$$h = 20 \tan 60^\circ = 20 \times \sqrt{3}$$

$$= 20\sqrt{3} \text{ m.}$$

Q.2 A circus artist is climbing from ground along a rope stretched from the top of a vertical pole and tied at the ground. The height of the pole is 12 m and the angle made by the rope with ground level is 30°. Calculate the distance covered by the artist in climbing to the top of the pole.

Solution



We have to find AC = ?

$$\frac{AB}{AC} = \sin 30^\circ \Rightarrow \frac{12}{l} = \sin 30^\circ$$

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$$\Rightarrow \frac{12}{\sin 30^\circ} = l \Rightarrow l = 12 \times 2 = 24 \text{ m}$$

10,11,12,13

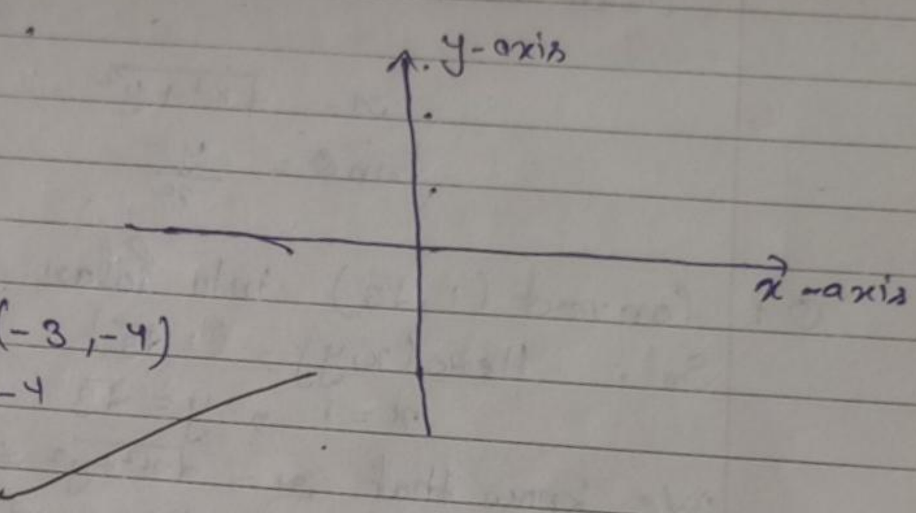
Page No. | Date

Ch = 10

Point

Definition of Point : It is an ordered pair of real number . for e.g (x, y) = (2, 4)

Q1 Plot the point.
(2, 5)
Sol.
x = 2 , y = 5



2 Plot the point (-3, -4)
Sol:; x = -3 , y = -4

(i) Conversion of Cartesion coordinate into Palar Coordinates

Cartesion coordinates $= (x, y)$
Palar coordinates $= (r, \theta)$

$$r = \sqrt{x^2+y^2}$$
$$\tan\theta = \frac{y}{x}$$

Q1 Convert $(1, \sqrt{3})$ into Palar coordinates.

Sol. Here $(x,y) = (1, \sqrt{3})$

$x = 1$, $y = \sqrt{3}$

We know that $r = \sqrt{x^2+y^2} = \sqrt{(1)^2+(\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{1}$ ✓

$\tan\theta = \sqrt{3}$

$\theta = 60^\circ = \frac{\pi}{3}$

Hence, Palar coordinates $(r, \theta) = \underline{\underline{\left(2, \frac{\pi}{3}\right)}}$

2 Convert $(-2, 2\sqrt{3})$ into Palar Coordinates

Sol. Here $(x,y) = (-2, 2\sqrt{3})$

$x = -2$, $y = 2\sqrt{3}$

We know that $r = \sqrt{x^2+y^2} = \sqrt{(-2)^2+(2\sqrt{3})^2} = \sqrt{4+12}$

$= \sqrt{16} = 4$

$\tan\theta = \frac{y}{x} = \frac{\cancel{2}\sqrt{3}}{-\cancel{2}} = -\sqrt{3}$ ✓

$\theta = 60^\circ = -\frac{\pi}{3}$

Hence, Palar coordinates $(r, \theta) = \left(4, -\frac{\pi}{3}\right)$

(ii) Conversion of Palar coordinates into cartesion coordinates

$x = r\cos\theta$
$y = r\sin\theta$

Q3. Convert $\left(2, \frac{\pi}{4}\right)$ into Cartesian coordinates.

Sol. $r = 2$, $\theta = \frac{\pi}{4}$

We know that

$x = 2\cos\frac{\pi}{4}$ $\qquad$ $y = 2\sin\frac{\pi}{4}$

$x = 2\cos 45^\circ$ $\qquad$ $y = 2\sin 45^\circ$

$x = 2 \times \frac{1}{\sqrt{2}}$ $\qquad$ $y = 2 \times \frac{1}{\sqrt{2}}$

$x = \frac{2}{\sqrt{2}} = \sqrt{2}$ $\qquad$ $y = \frac{2}{\sqrt{2}} = \sqrt{2}$ ✓

Hence $(x,y) = (\sqrt{2}, \sqrt{2})$

4 Convert $\left(3, -\frac{\pi}{6}\right)$ into Cartesion coordinates

Sol. $r = 3$, $\theta = -\frac{\pi}{6}$

We know that

$x = 3\cos -\frac{\pi}{6}$ $\qquad$ $y = 3\sin -\frac{\pi}{6}$

$x = 3\cos\frac{\sqrt{3}}{2}$ $\qquad$ $y = 3\sin -\frac{1}{2}$ ✓

$x = \frac{3\sqrt{3}}{2}$ $\qquad$ $y = \frac{-3}{2}$

Hence $(x, y) = \left(\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$

Distance formula :- It is used to find the distance between two points $P(x, y)$ and $Q(x_2, y_2)$

formula $Pq = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

1 find the distance between the fallowing points.

(i) $(3,5)$ and $(-1,4)$

Sol . $PQ = \sqrt{(-1-3)^2 + (4-5)^2}$

$= \sqrt{16+1} = \sqrt{17}$

(ii) $(-2,3)$ and $(-1,-4)$

Sol $PQ = \sqrt{(-1+2)^2 + (-4-3)^2}$

$= \sqrt{(1)^2 + (-7)^2}$

$= \sqrt{1+49} = \sqrt{50}$

(iii) $(1,0)$ and $(-1,5)$

Sol = $PQ = \sqrt{(-1-1)^2 + (5-0)^2}$

$= \sqrt{(2)^2 + (5)^2}$

$= \sqrt{4+25} = \sqrt{29}$

Mid-point :- It is used to find the mid-points of the line joining by points (x, y) and (x_2, y_2)

formula $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

1 find the mid-point of the fallowing line joining by points.

(i) $(3,5)$ and $(-6,4)$
(ii) $(2,4)$ and $(4,6)$
(iii) $(1,0)$ and $(3,4)$

Solution. (i) $(x, y_1) = (3,5)$ and $(x_2, y_2) = (-6,4)$

Mid-point is $\left(\frac{3-6}{2}, \frac{5+4}{2}\right)$

$= \left(\frac{-3}{2}, \frac{9}{2}\right)$

(ii) $(x_1, y_1) = (2,4)$, $(x_2, y_2) = (4,6)$

$= \left(\frac{2+4}{2}, \frac{4+6}{2}\right)$

$= \left(\frac{\cancel{6}}{\cancel{2}}, \frac{\cancel{10}}{\cancel{2}}\right) = (3, 5)$

(iii) $= (1,0)$ and $(3,4)$

$= \left(\frac{1+3}{2}, \frac{0+4}{2}\right)$

$= \left(\frac{\cancel{4}}{\cancel{2}}, \frac{\cancel{4}}{\cancel{2}}\right)$

$= (2, 2)$

Centroid of vertices of triangle :- If a triangle has three vertices (x,y) (x_2,y_2) and (x_3,y_3), then centroid of triangle is given by

$$\text{formula} = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Q1 Find the centroid of the triangle which has the following vertices.

(i) (2,0), (3,5) and (3,4)
(ii) (3,-5), (-1,-1) and (-2,-3)
(iii) (1,-1), (3,-4) and (-5,2)

Solution

(i) $(x,y_1)=(2,0)$, $(x_2,y_2)=(3,5)$, $(x_3 y_3)=(3,4)$

Centroid is $\left(\frac{2+3+3}{3}, \frac{0+5+4}{3}\right)$

$= \left(\frac{8}{3}, \frac{9}{3}\right) = \left(\frac{8}{3}, 3\right)$

(ii) $(x,y_1)=(3,-5)$, $(x_2 y_2)=(-1,-1)$, $(x_3,y_3)=(-2-3)$

Centroid is $\left(\frac{3-1-2}{3}, \frac{-5-1-3}{3}\right) = \cancel{\left(\frac{6}{3}\right.}, \left.\frac{7}{3}\right)$

$= \left(2, \frac{7}{3}\right)$

(iii) $(x_1,y_1)=(1,-1)$, $(x_2,y_2)=(3,-4)$, $(x_3,y_3)=(-5,2)$

Centroid is $\left(\frac{1+3-5}{3}, \frac{-1-4+2}{3}\right) = \left(\frac{-1}{3}, \frac{7}{3}\right)$

Point

Ch- 10

Ex- 10.1

① (-6,7) and (-1,-5)

Sol $PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$PQ = \sqrt{(-1+6)^2+(-5-7)^2}$

$PQ = \sqrt{(5)^2+(12)^2}$

$PQ = \sqrt{25+144}$

$PQ = \sqrt{169}$

$PQ = 13$

(3) (2,7) and (1,-8)

$PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$PQ = \sqrt{(1-2)^2+(-8-7)^2}$

$PQ = \sqrt{(1)^2+(15)^2}$

$PQ = \sqrt{1+225}$

$PQ = \sqrt{226}$

4) $(-1,-1)$, $(1,1)$, $(-\sqrt{3},\sqrt{3})$

Sol $\text{formula} = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$(x_1,y_1)=(-1,-1)$, $(x_2,y_2)=(1,1)$, $(x_3,y_3)=(-\sqrt{3},\sqrt{3})$

$= \left(\frac{-1+1-\sqrt{3}}{3}, \frac{-1+1+\sqrt{3}}{3}\right)$

$= \left(\frac{-\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

2) $(\overset{x_1}{b+c}, \overset{y_1}{c+a})$ and $(\overset{x_2}{c+a}, \overset{y_2}{a+b})$

Sol

$PQ = \sqrt{(\cancel{c}+a)^2 + (b+\cancel{c})^2 + (\cancel{a}+b)^2 - (c+\cancel{a})^2}$

$PQ = \sqrt{(a+b)^2 + (b-c)^2}$

$PQ = \sqrt{a^2+b^2-2ab+b^2+c^2-2bc}$

$PQ = \sqrt{a^2+2b^2+c-2ab-2bc}$

10 find the mid-point of the fallowing line joining by points.

(i) $(2,-5)$ and $(3,7)$

(ii) $(5,6)$ and $(-2,3)$

(iii) $(4,2)$ and $(4,6)$

Sol (i) $(2,-5)$ and $(3,7)$

$$\text{Mid-point} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{2+3}{2}, \frac{-5+7}{2}\right)$$

$$= \left(\frac{5}{2}, \frac{\cancel{2}}{\cancel{2}}\right)$$

$$= \left(\frac{5}{2}, 1\right)$$

(ii) $(5,6)$ and $(-2,3)$

$$\text{Mid-point} = \left(\frac{5-2}{2}, \frac{6+3}{2}\right)$$

$$= \left(\frac{3}{2}, \frac{9}{2}\right)$$

(iii) $(4,2)$ and $(4,6)$

$$\text{Mid-point} = \left(\frac{4+4}{2}, \frac{2+6}{2}\right)$$

$$= \left(\frac{\cancel{8}}{\cancel{2}}, \frac{8}{2}\right)$$

$$= (4,4)$$

Chapter-11

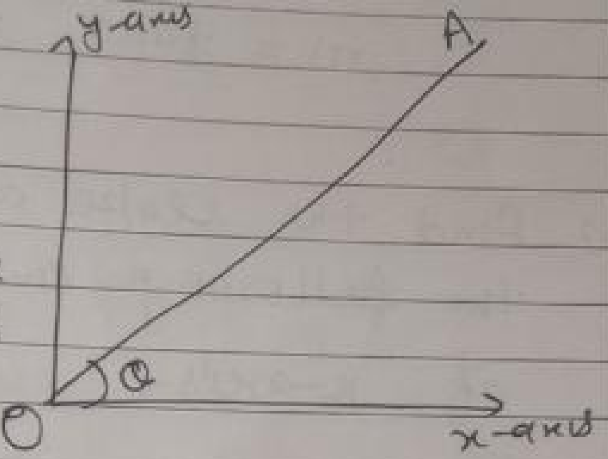
Straight line

The shortest distance between the two points is called straight line.

Slope of a line

$$\boxed{m = \tan\theta}$$ Imp

where θ is the angle made by line with ~~pos~~ x-axis



Note

1) slope of line parallel to x-axis is

$m = \tan\theta = \tan 0^\circ = 0$

slope of a line perpendicular to x-axis

$m = \tan 90^\circ$ = slope is not defined.

If a non-vertical line passes through two points (x_1, y_1), (x_2, y_2), then slope of line is $\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$ V.Imp

Q.1 Find the slope of a line which makes an angle of 60° with the positive direction of x-axis.

Solution. Here $\theta = 60°$

$m = \tan\theta = \tan 60° = \sqrt{3}$

Q.2 Find the slope of lines which makes the fallowing angle with positive direction of x-axis.

① 30° (ii) 120° (iii) 150°

Solution ① $m = \tan 30° = \frac{1}{\sqrt{3}}$

② $m = \tan 120° = \tan(180° - 60°)$
$= -\tan 60° = -\sqrt{3}$

$[\because \tan(180 - \theta) = -\tan\theta]$

③ $m = \tan 150°$
$= \tan(180 - 30°)$
$= -\tan 30° = \frac{-1}{\sqrt{3}}$

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Q.3 Find the slope of line passing through the points (2,3) and (−4,5).

Solution. $(x_1, y_1) = (2,3)$, $(x_2, y_2) = (-4,5)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{-4-2} = \frac{2}{-6} = \frac{1}{-3}$$ Ans

Q.4 Find the slope of a line passing through the points

① (4,5) and (−2,4)

② (−4,−2) and (−3,−1)

Solution. ① $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-5}{-2-4} = \frac{-1}{-6} = \frac{1}{6}$

② $m = \frac{-1+2}{-3+4} = \frac{1}{1} = 1$

Q.5 Find the value of x if the slope of line joining (0, x) and (2, 3x) is 5

Solution.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$5 = \frac{3x - x}{2 - 0}$$

$$5 = \frac{2x}{2}$$

$$5 = x$$ Ans

Q.6 Prove the line joining A (2,4), B(6,8) and C(1,3) and (3,5) are parallel.

Solution. Two lines are parallel if their slopes are equal. i.e. $m_1 = m_2$

m_1 = slope of line AB $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-4}{6-2} = \frac{4}{4} = 1$

m_2 = slope of line CD $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{3-1} = \frac{2}{2} = 1$

$\Rightarrow$ slope of line AB = slope of CD that is $m_1 = m_2$

$\Rightarrow$ lines AB and CD are parallel.

Q.7 Prove that the line joining A (2,3), B(4,5) and C(4,2), D(3,3) are perpendicular.

Solution. Two lines are perpendicular if product of their slope is −1

i.e. $m_1 \cdot m_2 = -1$

m_1 = slope of AB $= \frac{5-3}{4-2} = \frac{2}{2} = 1$

m_2 = slope of line CD $= \frac{3-2}{3-4} = \frac{1}{-1} = -1$

Now $m_1 \cdot m_2 = 1 \times -1 = -1$

$\Rightarrow$ lines AB and CD are perpendicular.

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Q.8 Find the value of 'x' for which the point (x, 2), (1, 3) and (4, 5) are collinear.

Solution.

Let the given points A (x, 2), B (1, 3) and C (4, 5).

The three points are said to be collinear

slope of AB = slope of BC

$$\frac{3-2}{1-x} = \frac{5-3}{4-1}$$

$$\frac{1}{1-x} = \frac{2}{3}$$

$$3 = 2(1-x)$$

$$3 = 2-2x \Rightarrow 2x = 2-3$$

$$2x = -1$$

$$x = -1/2 \text{ Ans.}$$

C.W.

Q.1 Find the slope of a line which makes the fallowing angle with positive direction of x-axi's

① 45°   (ii) 30°   (ii) 135°

Q.2 Find the slope of line passing through the points

① (2, 4) and (7, 6)

② (1, −2) and (−4, 3)

Q.3 Find the value of x, if the slope of line joining $(0, x)$ and $(-1, 2x)$ is 4.

Q.4 Find the value of x, if ~~line joining~~ the points $(x, 2)$, $(1, 3)$ and $(4, 5)$ are callinear.

Q Equation of line

(1) Point slope Form :- To find the equation of the straight line having slope m and one point (x_1, y_1)

$$y - y_1 = m(x - x_1)$$

Q.1 Find the equation of the line passing through (2,3) and having slope 5.

Solution. $(x_1, y_1) = (2, 3)$, $m = 5$

Equation of line is $y - y_1 = m(x - x_1)$

$$y - 3 = 5(x - 2)$$

$$y - 3 = 5x - 10$$

$$5x - 10 - y + 3 = 0$$

$$5x - y - 7 = 0$$ Ans.

Q.2 Find the equation of the line passing through the point (-1,2) and having slope 3.

Solution. $(x_1, y_1) = (-1, 2)$, $m = 3$

Equation of line is $y - y_1 = m(x - x_1)$

$$y - 2 = 3(x + 1)$$

$$y - 2 = 3x + 3$$

$$3x + 3 - y + 2 = 0$$

$$3x - y + 5 = 0$$ Ans.

Q.3 Find the equation of line passing through the point (4, -3) and makes an angle 120° with positive direction of x-axis

Solution. $(x_1, y_1) = (4, -3)$

$m = \tan\theta = \tan 120°$

$= \tan(180-60) = -\tan 60°$

$= -\sqrt{3}$

Equation of line is

$y - y_1 = m(x - x_1)$

$y + 3 = -\sqrt{3}(x-4)$

$y + 3 = -\sqrt{3}x + 4\sqrt{3}$

$y + 3 + \sqrt{3}x - 4\sqrt{3} = 0$

$\sqrt{3}x + y + 3 - 4\sqrt{3} = 0$ Ans

② Equation of line (Two-Point Form)

② when the line passing through two points (x_1, y_1) and (x_2, y_2),

then Equation of line is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

OR

$$y - y_1 = m(x - x_1)$$

Where $m = \frac{y_2 - y_1}{x_2 - x_1}$

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Q.1 Find the equation of line passing through the points $(-1, 2)$ and $(3,4)$

Solution. $(x_1, y_1) = (-1, 2)$, $(x_2, y_2) = (3, 4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{3+1} = \frac{2}{4} = \frac{1}{2}$$

Equation of line is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{2}(x+1)$$

$$2(y-2) = x+1$$

$$2y - 4 = x+1 \Rightarrow x+1-2y+4=0$$

$$x - 2y + 5 = 0$$ Ans

Q.2 Find the equation of line passing through the points $(0,-1)$ and $(2,-3)$

Solution. Here $(x_1, y_1) = (0,-1)$, $(x_2, y_2) = (2,-3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3+1}{2-0} = \frac{-2}{2} = -1$$

Equation of line is

$$y - y_1 = m(x - x_1)$$

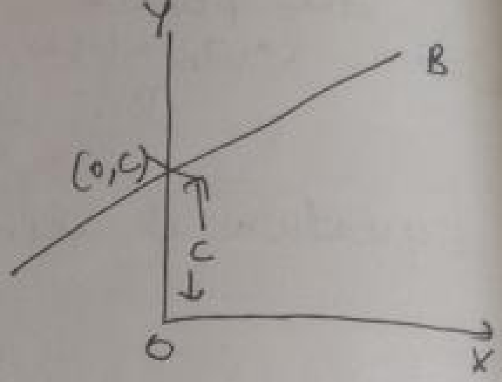
$$y+1 = m(x-0)$$

$$y+1 = (-1)(x)$$

$$y+1 = -x$$

$$y+x+1 = 0$$ Ans

③ Slope Intercept Form :- To find the equation of straight line whose slope m and y-intercept 'c' are given.



Equation af line is

$$y = mx + c$$

Note ① If intercept 'c' is above x-axis, then it is positive

② If intercept 'c' is below x-axis, then it is negative.

Q.1 Find the equation of a line having slope 5 and cut off intercept 7 from the positive direction of y-axis

Solution. Here, $m = 5$, $c = 7$

Equation af line is $y = mx + c$

$$y = 5x + 7$$

$$5x - y + 7 = 0$$ Ans.

Q.2 Find the equation of a line having slope 2 and cutoff intercept 5 from negative direction of y-axis

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Solution Here $m = 2$, $c = -5$

Equation of line is $y = mx + c$

$y = 2x \mp 5$

$2x - y \mp 5 = 0$ Ans.

Q.3 Find the equation of a line which makes an angle 60° with positive direction of x-axis and cutoff intercept 3 from the positive direction of y-axis.

Solution.

Here $m = \tan\theta = \tan 60^\circ = \sqrt{3}$

$c = 3$

Equation of line is $y = mx + c$

$y = \sqrt{3}x + 3$

$\sqrt{3}x - y + 3 = 0$ Ans.

④ Intercept Form

Equation of line is $\frac{x}{a} + \frac{y}{b} = 1$

where a = intercept on x-axis

b = intercept on y-axis.

Q.1 Find the equation of a line which makes 3 and 4 intercepts on the axes

Solution. Here $a = 3$, $b = 4$

Equation of line is $\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{3} + \frac{y}{4} = 1 \Rightarrow \frac{4x+3y}{12} = 1$

$\Rightarrow 4x + 3y = 12 \Rightarrow 4x + 3y - 12 = 0$

Ans.

C.W.

Q.1 Find the equation of line having slope 5 and passes through the point (2,3).

Q.2 Find the equation of line passing through (-1,3) and makes an angle 135° with positive direction of x-axis.

Q.3 Find the equation of line passing through the points (1,2) and (4,-3).

Q.4 Find the equation of line having slope 5 and cut off intercept 4 from the positive direction of y-axis.

Q.5 Find the equation of line which makes an angle 45° with x-axis and cut off intercept 7 from the positive direction of y-axis.

Q.6 Find the equation of line having slope 2 and cut-off intercept 6 from the negative direction of y-axis.

Q.7 Find the equation of line which makes -2 and 5 intercepts on coordinates axis respectively.

Q.8 Find the equation of line having $p = 4$ and $\alpha = 150°$

Q.2 Find the equation of line cutting off intercepts a and b where $a = 7$, $b = -3$.

Solution- Here $a = 7$, $b = -3$

Equation of line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{7} + \frac{y}{-3} = 1$$

$$\frac{x}{7} - \frac{y}{3} = 1$$

$$\frac{3x - 7y}{21} = 1$$

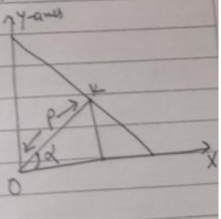
$$3x - 7y = 21$$

$$3x - 7y - 21 = 0$$ Ans.

⑤ Normal Form or Perpendicular Form

Equation of line is

$$p = x \cos\alpha + y \sin\alpha$$



Where p = pendicular distance of the line from the origin

and α is the angle made by perpendicular with x-axis

Angle between two-lines.

$$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right|$$

where = m_1 = slope of line ①

m_2 = slope of line ②

If $ax + by + c = 0$ is the equation of straight line.

$$ax + by + c = 0$$
$$by = -ax - c$$
$$y = -\frac{a}{b}x - \frac{c}{b}$$
$$y = mx + c$$

where $m = -\frac{a}{b}$, $c = \frac{-c}{b}$

$$m = -\frac{\text{coefficient of } x}{\text{coefficient of } y}$$

Q. 1 Find the acute angle between the lines whose slopes are 3 and $1/2$

Solution. $m_1 = 3$, $m_2 = 1/2$

Now $\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right|$

$$\tan\theta = \left| \frac{3 - \frac{1}{2}}{1 + (3)\left(\frac{1}{2}\right)} \right|$$

$$\tan\theta = \left|\frac{3-\frac{1}{2}}{1+\frac{3}{2}}\right|$$

$$\tan\theta = \left|\frac{\frac{6-1}{2}}{\frac{2+3}{2}}\right| = \left|\frac{\cancel{\frac{5}{2}}}{\cancel{\frac{5}{2}}}\right| = 1$$

$$\tan\theta = 1$$

$$\theta = 45^\circ = 45^\circ \times \frac{\pi}{180} = \frac{\pi}{4}$$

Q.2. Find the angle between the lines whose equations are

$$\sqrt{3}x + y - 1 = 0 \quad , \quad x + \sqrt{3}y - 1 = 0$$

Solution. Here $\sqrt{3}x + y - 1 = 0$, $x + \sqrt{3}y - 1 = 0$

$$m_1 = \frac{-\text{coefficent of } x}{\text{coefficent of } y} = \frac{-\sqrt{3}}{1} = -\sqrt{3}$$

$$m_2 = \frac{-1}{\sqrt{3}}$$

Now $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 \cdot m_2}\right| = \left|\frac{-\sqrt{3} + \frac{1}{\sqrt{3}}}{1 + (-\cancel{\sqrt{3}})\left(\frac{-1}{\cancel{\sqrt{3}}}\right)}\right|$

$$\tan\theta = \left|\frac{\frac{-3+1}{\sqrt{3}}}{1+1}\right|$$

$$\tan\theta = \left|\frac{\frac{-2}{\sqrt{3}}}{2}\right| = \left|\frac{-2}{\sqrt{3}} \times \frac{1}{2}\right| = \left|\frac{-1}{\sqrt{3}}\right|$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

Q3 Find the angle between the lines whose equation are

$$(2-\sqrt{3})x - y + 5 = 0 \quad \text{and} \quad (2+\sqrt{3})x - y - 7 = 0$$

Solution

Here $(2-\sqrt{3})x - y + 5 = 0$, $(2+\sqrt{3})x - y - 7 = 0$

$$m_1 = -\frac{\text{coeffcient of } x}{\text{coeffcient of } y} = \frac{-(2-\sqrt{3})}{-1} = 2-\sqrt{3}$$

$$m_2 = \frac{-(2+\sqrt{3})}{-1} = 2+\sqrt{3}$$

Now $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$\tan\theta = \left|\frac{(2-\sqrt{3}) - (2+\sqrt{3})}{1 + (2-\sqrt{3})(2+\sqrt{3})}\right|$$

$$= \left|\frac{\cancel{2} - \sqrt{3} - \cancel{2} - \sqrt{3}}{1 + (2^2 - (\sqrt{3})^2)}\right| = \left|\frac{-2\sqrt{3}}{1+4-3}\right|$$

$$= \left|\frac{-2\sqrt{3}}{2}\right| = |-\sqrt{3}| = \sqrt{3}$$

$$\tan\theta = \sqrt{3} \Rightarrow \theta = 60^\circ$$

C·W

Q.1 Find the acute angle between the lines whose slopes are

0 and 1

Q.2 Find the obtuse angle between the lines whose slopes are

$\frac{1}{\sqrt{3}}$ and $\frac{-1}{\sqrt{3}}$

Q.3 Find the angles between the pair of lines whose equation are

$2x-y+1=0$ and $x+y+8=0$

Q.4 The acute angle between two line is $\frac{\pi}{4}$. and slope of one of them is -3. Find the slope of the other line.

Teacher's Signature : ....

Q.4 The acute angle between two line is $\frac{\pi}{4}$ and slope of one of them $1/2$. Find the slope of the ~~lir~~ other line.

Solution. Here $\theta = \frac{\pi}{4} = 45'$

$$m_1 = 1/2 \quad , \quad m_2 = ?$$

Now $$\tan\theta = \left|\frac{m_1 - m_2}{1+m_1m_2}\right|$$

$$\tan 45' = \left|\frac{\frac{1}{2} - m_2}{1+\left(\frac{1}{2}\right)m_2}\right|$$

$$1 = \left|\frac{\frac{1-2m_2}{2}}{1+\frac{m_2}{2}}\right| = \left|\frac{\frac{1-2m_2}{2}}{\frac{2+m_2}{2}}\right|$$

$$1 = \left|\frac{1-2m_2}{2+m_2}\right|$$

Case I $$\frac{1-2m_2}{2+m_2} = 1$$

$$1-2m_2 = 2+m_2$$

$$1-2 = m_2+2m_2$$

$$-1 = 3m_2$$

$$-1/3 = m_2$$

Case II $$\frac{1-2m_2}{2+m_2} = -1$$

$$1-2m_2 = -1(2+m_2)$$

$$1-2m_2 = -2-m_2$$

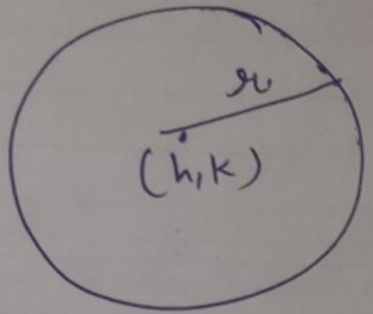
$$-2m_2+m_2 = -2-1$$

$$-m_2 = -3$$

$$m_2 = 3$$ Ans

chapter-12

Equation of circle.



Standard Equation of circle

$$(x-h)^2+(y-k)^2=r^2$$

To find the equation of circle whose centre and radius are given

centre (h,k), radius $= r$

Equation is

$$(x-h)^2+(y-k)^2=r^2$$

Q.1 find the equation of circle whose centre is (2,3) and radius 5

Solution. Equation of circle is

$$(x-h)^2+(y-k)^2=r^2$$

$$(h,k)=(2,3),\quad r=5$$

$$(x-2)^2+(y-3)^2=(5)^2$$

$$x^2+4-4x+y^2+9-6y=25$$

$x^2 + y^2 - 4x - 6y + 13 = 25$

$x^2 + y^2 - 4x - 6y + 13 - 25 = 0$

$x^2 + y^2 - 4x - 6x - 12 = 0$ Ay.

3.2 find the equation of circle whose centre is (-3, 4) and radius is $\sqrt{7}$

lution. Here $(h, k) = (-3, 4)$, $r = \sqrt{7}$.

Equation of circle is

$(x-h)^2 + (y-k)^2 = (r)^2$

$(x+3)^2 + (y-4)^2 = (\sqrt{7})^2$

$x^2 + 9 + 6x + y^2 + 16 - 8y = 7$

$x^2 + y^2 + 6x - 8y + 25 - 7 = 0$

$x^2 + y^2 + 6x - 8y + 18 = 0$

Q.3 Find the equation of circle having centre $(-1,-2)$ and radius $2\sqrt{5}$

Solution Here $(h,k) = (-1,-2)$, $r = 2\sqrt{5}$

$$(x+1)^2 + (y+2)^2 = (2\sqrt{5})^2$$

$$x^2 + 1 + 2x + y^2 + 4 + 4y = 20$$

$$x^2 + y^2 + 2x + 4y + 5 = 20$$

$$x^2 + y^2 + 2x + 4y + 5 - 20 = 0$$

$$x^2 + y^2 + 2x + 4y - 15 = 0$$ Ans

2marks General Equation of circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

where centre is $(-g, -f)$

radius $= \sqrt{g^2 + f^2 - c}$

V.Imp 4marks

Q.1 Find the centre and radius of equation of circle $x^2+y^2+4x+6y-3=0$

Salution Equation of circle is

$$x^2+y^2+4x+6y-3=0 \quad ①$$

comparing it with $x^2+y^2+2gx+2fy+c=0$

$2g = 4$, $2f = 6$, $c = -3$

$g = \frac{4}{2}$, $f = \frac{6}{2}$

$g = 2$, $f = 3$, $c = -3$

centre is $(-g, -f)$

$= (-2, -3)$

radius $= \sqrt{g^2+f^2-c} = \sqrt{(2)^2+(3)^2-(-3)}$

$= \sqrt{4+9+3} = \sqrt{16} = 4$

2 Find the centre and radius of equation of $x^2+y^2-4x-8y+7=0$

lution.
Given equation of circle is

$$x^2+y^2-4x-8y+7=0$$

comparing it with $x^2+y^2+2gx+2fy+c=0$

$2g = -4$, $2f = -8$, $c = 7$

$g = \frac{-4}{2}$, $f = \frac{-8}{2}$, $c = 7$

$g = -2$ $f = -4$

centre $= (2, 4)$

radius $= \sqrt{g^2+f^2-c} = \sqrt{(2)^2+(4)^2-7}$

$= \sqrt{4+16-7} = \sqrt{13}$

Q.3 Find the centre and radius of circle $x^2+y^2-4x+12y-3=0$

Salution.
The given equation of circle is

$x^2+y^2-4x+12y-3=0$

Comparing it with $x^2+y^2+2gx+2fy+c=0$

$2g = -4$
$g = \frac{-4}{2} = -2$

$2f = 12$
$f = \frac{12}{2} = 6$

$c = -3$

centre is $= (-g, -f)$
$= (2, -6)$

radius $= \sqrt{g^2+f^2-c}$
$= \sqrt{(2)^2+(-6)^2-(-3)}$
$= \sqrt{4+36+3} = \sqrt{43}$ Ans.

Q.4 Find the centre and radius of equation of circle
$$2x^2+2y^2+12x-8y+5=0$$

Salution.
$$2x^2+2y^2+12x-8y+5=0 \quad \text{—} ①$$

Divide ① by 2
$$x^2+y^2+6x-4y+\frac{5}{2}=0 \quad \text{—} ②$$

comparing it with $x^2+y^2+2gx+2fy+c=0$

$2g=6$	$2f=-4$	$c=5/2$
$g=\frac{6}{2}$	$f=-\frac{4}{2}$	
$g=3$	$f=-2$	

centre is $= (-g, -f) = (-3, 2)$

radius $= \sqrt{g^2+f^2-c} = \sqrt{(-3)^2+(2)^2-5/2}$

$= \sqrt{9+4-5/2} = \sqrt{13-\frac{5}{2}} = \sqrt{\frac{26-5}{2}} = \sqrt{\frac{21}{2}}$ Ans.

Imp

Equation of circle in diameteric farm.

To find the equation of a circle when end points of a diameter are given

Let (x_1, y_1) and (x_2, y_2) be end point of diameter, then Equation of circle is
$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

Q.1 Find the equation of circle if (2,3) and (−1,4) are the end points of a diameter of circle.

Salution.
Here $(x_1, y_1) = (2,3)$, $(x_2, y_2) = (-1,4)$

Equation of circle is
$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$
$$(x-2)(x+1)+(y-3)(y-4)=0$$
$$x^2-2x+x-2+y^2-3y-4y+12=0$$
$$x^2+y^2-x-7y+10=0$$ Ans.

Q.2 Determine the equation of circle if (2,−4) and (−1,−5) are the end points of diameter.

Salution.
Here $(x_1, y_1) = (2, -4)$
$(x_2, y_2) = (-1, -5)$

Equation of circle is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

$$(x-2)(x+1)+(y+4)(y+5)=0$$

$$x^2-2x+x-2+y^2+4y+5y+20=0$$

$$x^2+y^2-x+9y+18=0$$ Ans

8 marks

Q.1 Find the equation of circle passing through the points (1,2), (3,4) and (−2,3)

Solution. Let the equation of circle is

$$x^2+y^2+2gx+2fy+c=0 \quad \text{———} ①$$

Eq. ① passes through (1,2)

$$(1)^2+(2)^2+2g(1)+2f(2)+c=0$$

$$1+4+2g+4f+c=0$$

$$2g+4f+c+5=0 \quad \text{———} ②$$

Eq. ① passes through (3,4), we have

$$(3)^2+(4)^2+2g(3)+2f(4)+c=0$$

$$9+16+6g+8f+c=0$$

$$6g+8f+c+25=0 \quad \text{———} ③$$

Equation ① passes through (−2,3), we have

$$(-2)^2+(3)^2+2g(-2)+2f(3)+c=0$$

$$-4g+6f+c+4+9=0$$

$$-4g+6f+c+13=0 \quad \text{———} ④$$

Subtract ③ from ②,

$$2g+4f+c+5=0$$

$$\underline{\underset{-}{6g} \pm 8f \pm c \pm 25=0}$$

$$-4g-4f-20=0$$

$$-4(g+f+5)=0$$

$$g+f+5=0 \quad \text{———} ⑤$$

Subtract ④ from ③

$$6g+8f+c+25=0$$

$$\underline{\underset{+}{-4g} \underset{-}{+} 6f \underset{-}{+} c \underset{-}{+} 13=0}$$

$$10g+2f+12=0$$

$$2(5g+f+6)=0$$

$$5g+f+6=0 \quad \text{———} ⑥$$

Subtract ⑥ from ⑤

$$g+f+5=0$$

$$\underline{\underset{-}{5g} \pm f \pm 6=0}$$

$$-1=0$$

$$\Rightarrow -4g=1$$

$$g=1/-4$$

Put $g = -\frac{1}{4}$ ~~from~~ in ⑤

$-\frac{1}{4} + f + 5 = 0$

$f + \frac{5}{1} - \frac{1}{4} = 0 \Rightarrow f + \frac{19}{4} = 0$

$f = -\frac{19}{4}.$

Put $g = -\frac{1}{4}$, $f = -\frac{19}{4}$ in ②

$2\left(-\frac{1}{4}\right) + 4\left(-\frac{19}{4}\right) + c + 5 = 0$

$-\frac{1}{2} - 19 + c + 5 = 0$

$-\frac{1}{2} - 14 + c = 0 \Rightarrow -\frac{29}{2} + c = 0$

$c = \frac{29}{2}$

Put $g = -\frac{1}{4}$, $f = -\frac{19}{4}$, $c = \frac{29}{2}$ in ①

$x^2 + y^2 + 2\left(-\frac{1}{4}\right)x + 2\left(-\frac{19}{4}\right)y + \frac{29}{2} = 0$

$x^2 + y^2 - \frac{x}{2} - \frac{19}{2}y + \frac{29}{2} = 0$

$2x^2 + 2y^2 - x - 19y + 29 = 0$ Ans.

13

INTRODUCTION TO MATLAB

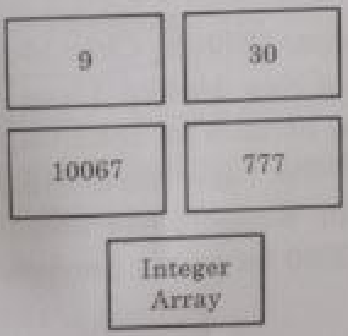
MATLAB

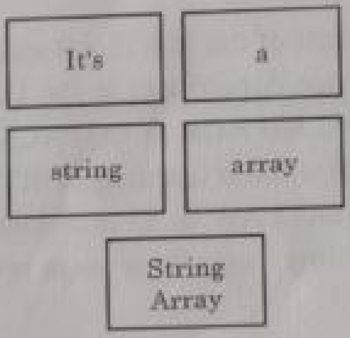
MATLAB stands for Matrix Laboratory. It is a high-performance language that is used for technical computing. It provides an interactive environment with hundreds of built-in functions for technical computing, graphics, and animations. It was developed by Cleve Molar of the company Math Works. Inc in the year 1984. It is written in C, C++, Java. It allows matrix manipulations, plotting of functions, and implementation of algorithms and creation of user interfaces.

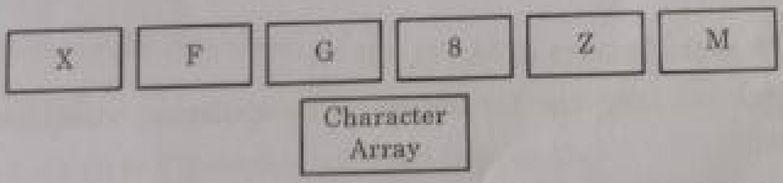
MATLAB is a programming and numeric computing platform used by millions of engineers and scientists to analyze data, develop algorithms, and create models.

MATLAB is also a programming language.

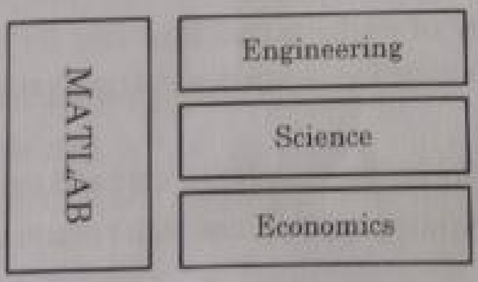
As its name contains the word Matrix, MATLAB does its' all computing based on mathematical matrices and arrays. MATLAB's all types of variables hold data in the form of the array only, let it be an integer type, character type or String type variable.



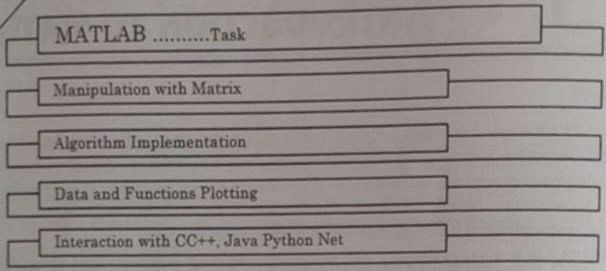




MATLAB is used in various disciplines of engineering, science, and economics.



MATLAB allows several types of tasks, such as manipulations with matrix, algorithm implementation, data, and functions plotting, and can interact with programs written in other programming languages.



MATLAB is a dynamic and weakly typed programming language.

MATLAB environment handles tasks of the declaration of the data type of the variables and provision for an appropriate amount of storage for the variables.

History of MATLAB

The development of the MATLAB started in the late 1970s by Cleve Moler, the chairman of the Computer Science department at the University of New Mexico. Cleve wanted to make his students able to use LINPACK & EISPACK (software libraries for numerical computing, written in FORTRAN), and without learning FORTRAN. In 1984, Cleve Moler with Jack Little & Steve Bangert rewrote MATLAB in C and founded Math Works. These libraries were known as JACKPAC at that time, later these were revised in 2000 for matrix manipulation and named as LAPACK.

Features of MATLAB

Following are the basic features of MATLAB –

- It is a high-level language for numerical computation, visualization and application development.
- It also provides an interactive environment for iterative exploration, design and problem solving.
- It provides vast library of mathematical functions for linear algebra, statistics, Fourier analysis, filtering, optimization, numerical integration and solving ordinary differential equations.
- It provides built-in graphics for visualizing data and tools for creating custom plots.
- It provides tools for building applications with custom graphical interfaces.

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Uses of MATLAB

MATLAB is widely used as a computational tool in science and engineering encompassing the fields of physics, chemistry, mathematics and all engineering streams. It is used in a range of applications including –

- Signal Processing and Communications
- Image and Video Processing
- Control Systems
- Test and Measurement
- Computational Finance
- Computational Biology

MATLAB's Power of Computational Mathematics

MATLAB is used in every facet of computational mathematics. Following are some commonly used mathematical calculations where it is used most commonly –

- Dealing with Matrices and Arrays
- 2-D and 3-D Plotting and graphics
- Linear Algebra
- Algebraic Equations
- Non-linear Functions
- Statistics
- Data Analysis
- Calculus and Differential Equations
- Numerical Calculations
- Integration
- Transforms
- Curve Fitting
- Various other special functions

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Advantages of MATLAB

1. Easy to use – You can learn this software very easily.
2. Huge community – It has huge community support, you go and visit Mathworks website there you will find many simulations.
3. Platform independent
4. Pre-defined functions reduce engineers' time.
5. Device-independent plotting
6. A graphical user interface, it will support for GUI.
7. MATLAB compiler

Disadvantages Of MATLAB

1. The speed of MAT-LAB is slow, you can overcome this by properly structuring the MAT-LAB program.
2. It is not open-source software

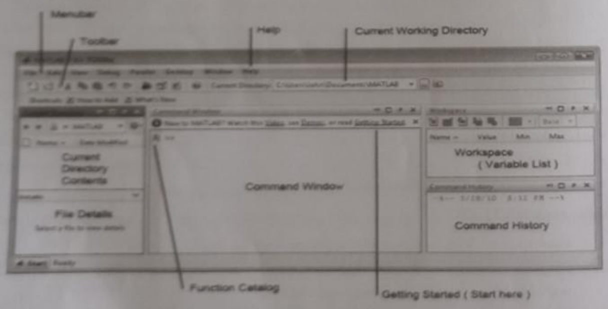
Alternatives To MATLAB

We all know that MAT-LAB is famous and good software, but the cost of MATLAB software is high. The other alternatives to MAT-LAB are Octave, **Scilab**, and Freemat.

Starting MATLAB

After logging into your account, you can enter MATLAB by double-clicking on the MATLAB shortcut icon (MATLAB 7.0.4) on your Windows desktop. When you start MATLAB, a special window called the MATLAB desktop appears. The desktop is a window that contains *other* windows. The major tools within or accessible from the desktop are:

- The Command Window
- The Command History
- The Workspace
- The Current Directory
- The Help Browser



A typical MATLAB Desktop

Command Window:

The Command Window is where you type in Matlab commands and expressions at the prompt (») and where the output of your command will be displayed. Command window is used to

- Execute commands
- Open the windows
- Run the programs
- Manage the MATLAB software

Workspace Browser:

- The Workspace shows the list of variables that are currently defined in a work session, and other information about the variables (type, size, value, etc.). It keeps track of variables you have defined as you execute the commands.
- Note: If any of the variables in the Workspace are plottable, they may be plotted quickly and easily by right-clicking on the variable name and selecting a plot type.
- Double click on a variable in the Browser and the Array Editor gets launched, which can be used to obtain information and sometimes edit properties of the variable.

Current Directory:

The Current Directory window, as its name suggests, displays the contents of the current working directory.

- Double click on any file to open it.
- Right-click on Matlab scripts and function files to execute the commands contained therein.
- Right-click on data files to import the data as Matlab variables.

Current directory can be changed by clicking on folders or use the **Current Working Directory** text box at the top of the Matlab working environment.

Command History:

The Command History window contains a list of commands, a user has entered recently in the Command Window, including both current and previous Matlab sessions.

- To execute a command in command history, double click on the command
- A command can be dragged to the command window and modified before execute again
- The entries in the command history can be removed by selecting the command and press 'Delete'
- To delete entire command history, Right click and select Clear Command history.
- These windows may be re-arranged according to your personal preferences, including dragging windows away from the Matlab work environment. In addition to these sub windows there are other special windows used to for special purposes.

Help Window:

The Help Window contains help information. This window can be opened from the **Help** menu in the toolbar of any MATLAB window. The Help Window is interactive and can be used to obtain information on any feature of MATLAB.

A **Start button** is provided on the lower left side, which is used to access MATLAB tools and features.

MATLAB Basics:

- **The Basic Features:**

Let us start with something simple, like defining a row vector with components the numbers 1, 2, 3, 4, 5 and assigning it a variable name, say x.

```
» x = [1 2 3 4 5]
x =
1 2 3 4 5
```

To create a column vector (MATLAB distinguishes between row and column vectors, as it should) we can either use semicolons (;) to separate the entries, or first define a row vector and take its transpose to obtain a column vector. Let us demonstrate this by defining a column vector y with entries 6, 7, 8, 9, 10 using both techniques.

```
» y = [6;7;8;9;10]
y =
6
7
8
9
10
» y = [6,7,8,9,10]
y =
    6  7  8  9  10
» y'
ans =
6
7
8
9
10
```

Let us make a few comments. First, note that to take the transpose of a vector (or a matrix for that matter) we use the single quote ('). Also note that MATLAB repeats (after it processes) what we typed in. Sometimes, however, we might not wish to "see" the output of a specific command. We can suppress the output by using a semicolon (;) at the end of the command line. Finally, keep in mind that MATLAB automatically assigns the variable name ans to anything that has not been assigned a name. In the example above, this means that a new variable has been created with the column vector entries as its value. The variable ans, however, gets recycled and every time we type in a command without assigning a variable, ans gets that value.

Using MATLAB as a calculator

MATLAB has all of the basic arithmetic operations built in:

+ addition
- subtraction
* multiplication
/ division (a/b = "a divided by b")
^ exponentiation

as well as many more complicated functions (e.g. trigonometric, exponential):

sin(x) sine of x (in radians)
cos(x) cosine of x (in radians)
exp(x) exponential of x
log(x) base e logarithm of x (normally written ln)

When working with arithmetic operations, it's important to be clear about the order in which they are to be carried out. This can be specified by the use of brackets. For example, if you want to multiply 5 by 2 then add 3, we can type

```
    (5*2)+3
ans =
    13
```

and we get the correct value. If we want to multiply 5 by the sum of 2 and 3, we type

```
    5*(2+3)
ans =
    25
```

and this gives us the correct value. Carefully note the placement of the brackets. If you don't put brackets, Matlab has its own built in order of operations: multiplication/division first, then addition/subtraction. For example:

```
    5*2+3
ans =
    13
```

gives the same answer as (5*2)+3. As another example, if we want to divide 8 by 2 and then subtract 3, we type

```
    (8/2)-3
ans =
    1
```

and get the right answer. To divide 8 by the difference between 2 and 3, we type

```
    8/(2-3)
ans =
    -8
```

and again get the right answer. If we type

```
    8/2-3
ans =
    1
```

we get the first answer - the order of operations was division first, then subtraction.

In general, it's good to use brackets - they invovle more typing, and may make a computation look more cumbersome, but they help reduce ambiguity regarding what you want the computation to do.

This is a good point to make a general comment about computing. Computers are actually quite stupid - they do what you tell them to, not what you want them to do. When you type any commands into a computer program like MATLAB, you need to be very careful that these two things match exactly.

You can always get help in MATLAB by typing "help". Type this alone and you'll get a big list of directories you can get more information about - which is not always too useful. It's more useful to type "help" with some other command.

Trigonometric Functions in MATLAB

Trigonometric functions are the mathematical functions that can result in the output with the given input.

There are six trigonometric functions –

1. Sine (sin)
2. Cosine(cos)
3. Tangent(tan)
4. CoTangent(cot)
5. Secant(sec)
6. CoSecant(csc)

Sine Function

sin: Sin function returns the sine of input in radians. The input can be a number or an array or a matrix.

Syntax: sin(value)

sind: This function returns the sine of input in degrees.

Syntax: sind(value)

asin: This function returns the inverse of sine in radians.

Syntax: asin(x)

asind: This function returns the inverse of sine in degrees.

Syntax: asind(x)

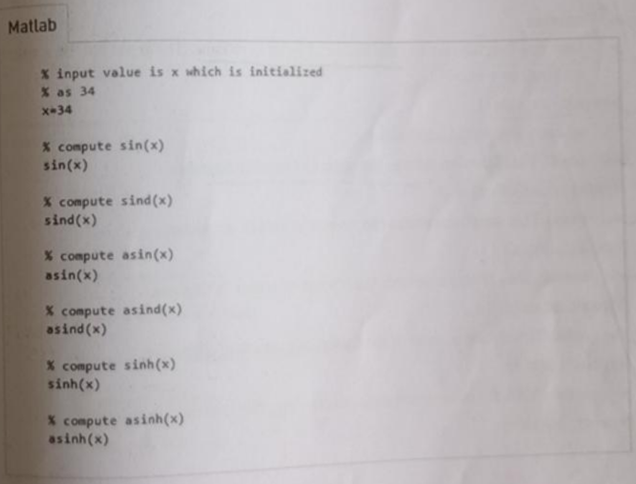
sinh: This function returns the hyperbolic sine of the value.

Syntax: sinh(x)

asinh: This function returns the inverse hyperbolic sine of the value.

Syntax: asinh(x)

Example:



```
% input value is x which is initialized
% as 34
x=34

% compute sin(x)
sin(x)

% compute sind(x)
sind(x)

% compute asin(x)
asin(x)

% compute asind(x)
asind(x)

% compute sinh(x)
sinh(x)

% compute asinh(x)
asinh(x)
```

Imp 8 marks

Differences Between MATLAB and Scilab :

S.No.	MATLAB	Scilab
1.	It is a high-level programming language that is used for performing mathematical computing.	It is a software that is used for performing scientific computations.
2.	MATLAB is short used for **Matrix laboratory.**	Scilab is short used for **Scientific Laboratory.**
3.	This language is written in C, C++, and Java.	This software is programmed with C, C++, and Fortran.
4.	The file saved is with extension **'geeksforgeeks.m'.**	The file saved is with extension **'geeksforgeeks.sci'.**
5.	The command line used begins with '%'.	The command line used beings with '//'.
6.	It is not an open-source language.	It is open-source software.
7.	MATLAB is used for solving high-level computation.	Scilab is used for solving low-level scientific computations.
8.	The syntax for declaring an empty matrix is [] in MATLAB.	The syntax for declaring an empty matrix is [] +1 in Scilab.

□□□

Infinity in Matlab → inf