Units and Dimensions

The standard quantity with which a physical quantity of the same kind is compared is called a **unit**.

Every measurement has two parts. The first is a number (n), and the next is a unit (u). Q = nu. For example, the length of an object = 40 cm. The number expressing the magnitude of a physical quantity is inversely proportional to the unit selected.

If n_1 and n_2 are the numerical values of a physical quantity corresponding to the units u_1 and u_2, then $\mathbf{n_1u_1 = n_2u_2}$. For example, 2.8 m = 280 cm; 6.2 kg = 6200 g.

Fundamental and Derived Quantities

- The quantities that are independent of other quantities are called **fundamental quantities**. The units that are used to measure these fundamental quantities are called **fundamental units**. There are four systems of units, namely CGS, MKS, FPS and SI.
- The quantities that are derived using the fundamental quantities are called **derived quantities**. The units that are used to measure these derived quantities are called **derived units**.

Fundamental and supplementary physical quantities in the SI system

Fundamental Quantity	**System of Units**		
	CGS	**MKS**	**FPS**
Length	centimeter	meter	foot
Mass	gram	kilogram	pound
Time	second	second	second

Physical Quantity	**Unit**	**Symbol**
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Intensity of light	candela	cd

Quantity of substance	mole	mol

Supplementary Quantities

Plane angle	Radian	rad
Solid angle	Steradian	sr

Most SI units are used in scientific research. SI is a coherent system of units.

A **coherent system** of units is one in which the units of derived quantities are obtained as multiples or submultiples of certain basic units. The SI system is a comprehensive, coherent and rationalised MKS.

Note:

- **Angstrom** is the unit of length used to measure the wavelength of light. 1 Å = 10^{-10} m.
- **Fermi** is the unit of length used to measure nuclear distances. 1 Fermi = 10^{-15} meter.
- A **light year** is the unit of length for measuring astronomical distances.
- **Light year** = distance travelled by light in 1 year = 9.4605 × 10^{15} m.
- **Astronomical unit** = Mean distance between the sun and earth = 1.5 × 10^{11} m.
- **Parsec** = 3.26 light years = 3.084×10^{16} m.
- **Barn** is the unit of area for measuring scattering cross-section of collisions. **1 barn = 10^{-28} m^2.**
- **Chronometer** and **metronome** are time-measuring instruments. The quantity having the same unit in all the systems of units is time.

Full names of the units, even when they are named after a scientist, should not be written with a capital letter. For example, newton, watt, ampere, meter. The unit should be written either in full or in agreed symbols only. Units do not take the plural form. For example, 10 kg but not 10 kgs, 20 w but not 20 ws. No full stop or punctuation mark should be used within or at the end of symbols for units. For example, 10 W but not 10 W.

Dimensions of a physical quantity are the powers to which the fundamental units are raised to obtain one unit of that quantity.

The expression showing the powers to which the fundamental units are to be raised to obtain one unit of a derived quantity is called the **dimensional formula** of that quantity.

If Q is the unit of a derived quantity represented by Q = $M^aL^bT^c$, then $M^aL^bT^c$ is called the dimensional formula, and the exponents a, b, and c are called dimensions.

The physical quantities with dimensions and a fixed value are called dimensional constants. For example, gravitational constant (G), Planck's constant (h), universal gas constant (R), velocity of light in a vacuum (C), etc.

Dimensionless quantities are those which do not have dimensions but have a fixed value.

- **Dimensionless quantities without units:** Pure numbers, π, e, sin θ, cos θ, tan θ etc.
- **Dimensionless quantities with units:** Angular displacement – radian, Joule's constant – joule/calorie, etc.

Dimensional variables are those physical quantities which have dimensions and do not have a fixed value. For example, velocity, acceleration, force, work, power, etc.

` **Dimensionless variables** are those physical quantities which do not have dimensions and do not have a fixed value. For example, specific gravity, refractive index, the coefficient of friction, Poisson's ratio, etc.

Principle of Homogeneity of Dimensions

In any correct equation representing the relation between physical quantities, the dimensions of all the terms must be the same on both sides. Terms separated by '+' or '–' must have the same dimensions.

Quantities having same Dimensions:

1. Impulse and momentum.
2. Work, torque, the moment of force, energy.
3. Angular momentum, Planck's constant, rotational impulse.
4. Stress, pressure, modulus of elasticity, energy density.
5. Force constant, surface tension, surface energy.
6. Angular velocity, frequency, velocity gradient.
7. Gravitational potential, latent heat.
8. Thermal capacity, entropy, universal gas constant and Boltzmann's constant.
9. Force, thrust.
10. Power, luminous flux.

Applications of Dimensional Analysis:

1. Verify the correctness of a physical equation.
2. Derive a relationship between physical quantities.
3. Converting the units of a physical quantity from one system to another system

Limitations of Dimensional Analysis:

1. Dimensionless quantities cannot be determined by this method. Also, the constant of proportionality cannot be determined by this method. They can be found either by experiment (or) by theory.
2. This method does not apply to trigonometric, logarithmic and exponential functions.
3. This method will be difficult in the case of physical quantities, which are dependent upon more than three physical quantities.
4. In some cases, the constant of proportionality also possesses dimensions. In such cases, we cannot use this system.
5. If one side of the equation contains the addition or subtraction of physical quantities, we cannot use this method to derive the expression.

Only similar physical quantities can be added or subtracted. Thus, two quantities having different dimensions cannot be added together. For example, we cannot add mass and force or electric potential and resistance.

For any given equation, the principle of homogeneity of dimensions is used to check the correctness and consistency of the equation. The dimensions of each component on either side of the sign of equality are checked, and if they are not the same, the equation is considered wrong.